Trigonometric vs. Hyperbolic - Comparisons

Note: The rows in yellow indicate the rule for hyperbolic is different than the rule for trigonometric.

Identities

Trigonometric	**Hyperbolic**
$\cos^2 x + \sin^2 x = 1$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2 \sin x \cos x$	$\sinh 2x = 2 \sinh x \cosh x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\cos^2 x = \frac{1+\cos 2x}{2}$	$\cosh^2 x = \frac{1+\cosh 2x}{2}$
$\sin^2 x = \frac{1-\cos 2x}{2}$	$\sinh^2 x = \frac{\cosh 2x-1}{2}$
$\tan^2 x = -1 + \sec^2 x$	$\tanh^2 x = 1 - \operatorname{sech}^2 x$
$\cot^2 x = -1 + \csc^2 x$	$\coth^2 x = 1 + \operatorname{csch}^2 x$

Note: Some people write cosec instead of csc, cosech instead of csch, and cotan instead of cot.

Derivatives

Trigonometric	**Hyperbolic**
$\frac{d}{dx} \sin x = \cos x$	$\frac{d}{dx} \sinh x = \cosh x$
$\frac{d}{dx} \cos x = -\sin x$	$\frac{d}{dx} \cosh x = \sinh x$
$\frac{d}{dx} \tan x = \sec^2 x$	$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$
$\frac{d}{dx} \cot x = -\csc^2 x$	$\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$
$\frac{d}{dx} \sec x = \sec x \tan x$	$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

Integrals

Trigonometric	Hyperbolic
$\int \sin x \; dx = -\cos x + C$	$\int \sinh x \; dx = \cosh x + C$
$\int \cos x \; dx = \sin x + C$	$\int \cosh x \; dx = \sinh x + C$
$\int \sec^2 x \; dx = \tan x + C$	$\int \operatorname{sech}^2 x \; dx = \tanh x + C$
$\int \csc^2 x \; dx = -\cot x + C$	$\int \operatorname{csch}^2 x \; dx = -\coth x + C$
$\int \sec x \tan x \; dx = \sec x + C$	$\int \operatorname{sech} x \tanh x \; dx = -\operatorname{sech} x + C$
$\int \csc x \cot x \; dx = -\csc x + C$	$\int \operatorname{csch} x \coth x \; dx = -\operatorname{csch} x + C$

Inverse Identities

Trigonometric	Hyperbolic
$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$	$\operatorname{sech}^{-1} x = \cosh^{-1}\left(\frac{1}{x}\right)$
$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$	$\operatorname{csch}^{-1} x = \sinh^{-1}\left(\frac{1}{x}\right)$
$\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$	$\coth^{-1} x = \tanh^{-1}\left(\frac{1}{x}\right)$

Note: Some people denote the inverses of these functions by arc; e.g. $\arcsin x$ stands for $\sin^{-1}(x)$, $\arccos x$ stands for $\cos^{-1}(x)$, etc.

Inverse Derivatives

Trigonometric	Hyperbolic
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}, \; x > 1$
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}, \; \lvert x\rvert < 1$
$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$	$\frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}, \; \lvert x\rvert > 1$
$\frac{d}{dx}\sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\operatorname{sech}^{-1} x = \frac{-1}{x\sqrt{1-x^2}}, \; 0 < x < 1$
$\frac{d}{dx}\csc^{-1} x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\operatorname{csch}^{-1} x = \frac{-1}{\lvert x\rvert\sqrt{1+x^2}}, \; x \neq 0$

Inverse Integrals

Trigonometric	Hyperbolic
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C,\ a > 0$
$\int \frac{dx}{\sqrt{a^2-x^2}} = -\cos^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C,\ x > a > 0$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$	$\int \frac{dx}{a^2-x^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & x^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & x^2 > a^2 \end{cases}$
$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{\lvert x\rvert}{a}\right) + C$	$\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{-1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C,\ 0 < x < a$
-	$\int \frac{dx}{x\sqrt{a^2+x^2}} = \frac{-1}{a}\operatorname{csch}^{-1}\left\lvert\frac{x}{a}\right\rvert + C,\ x \neq 0,\ a > 0$

Note: By taking $a = 1$ in the previous table, we get the following

Inverse Integrals for $a = 1$

Trigonometric	Hyperbolic
$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$	$\int \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x + C$
$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$	$\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x + C,\ x > 1$
$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$	$\int \frac{dx}{1-x^2} = \begin{cases} \tanh^{-1} x + C, & x^2 < 1 \\ \coth^{-1} x + C, & x^2 > 1 \end{cases}$
$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}\lvert x\rvert + C$	$\int \frac{dx}{x\sqrt{1-x^2}} = -\operatorname{sech}^{-1} x + C,\ 0 < x < 1$
-	$\int \frac{dx}{x\sqrt{1+x^2}} = -\operatorname{csch}^{-1}\lvert x\rvert + C,\ x \neq 0$

Function Definition

Trigonometric	**Hyperbolic**
$\sin x = \frac{e^{ix}-e^{-ix}}{2i}$	$\sinh x = \frac{e^{x}-e^{-x}}{2}$
$\cos x = \frac{e^{ix}+e^{-ix}}{2}$	$\cosh x = \frac{e^{x}+e^{-x}}{2}$

Trigonometric and Hyperbolic Function Relationships

$$\sinh x = -i \sin ix.$$

$$\cosh x = \cos ix.$$

We note here Euler's formula

$$e^{i\theta} = \cos\theta + i \sin\theta.$$