CSIT 241 - EXAM II

Name:

Instructions: Do all of the following. EXPLAIN every step. Points will be deducted for incomplete proofs or incomplete solutions. Do not use calculators. Do not look at your neighbor or talk to him. If you use a method different than what the question is asking for, you'll get no credit. Use only notation used in class. Any violation of the instructions may result in a partial credit or no credit at all.

The exam is closed book, closed notes. No material may be used and the Internet is not allowed.

Time: 50 minues.

(1) (3 points) Prove that the function $f : \mathbb{R}^+ \longrightarrow (1, e^2)$, defined by $f(x) = e^{(\frac{2x}{x+3})}$ is one-to-one.

(2) (3 points) Prove that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, defined by $f(x) = -5 + e^{(2x-7)^2}$ is not one-to-one.

(3) (3 points) Prove that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, defined by $f(x) = -5 + e^{(2x-7)^2}$ is not onto.

(4) (3 points) Let $f : \mathbb{R} \longrightarrow \mathbb{R}^+$, defined by $f(x) = e^{2x+7}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$, defined by $g(x) = x^2 + 8$. Find $g \circ f$ if defined.

(5) (5 points)Let $f: (\frac{5}{2}, \infty) \longrightarrow \mathbb{R}$, defined by $f(x) = \ln(2x - 5)$. Find f^{-1}.

(6) (3 points) Prove that the set $S = \{\frac{(-1)^n}{5n} \mid n \in \mathbb{N}\}$ is countable.

(7) (24 points) Deteemine if the following are true or fasle

(a) If $g \circ f$ is a bijection, then both f and g are bijections.

(b) If g and f are both invertible, then $g \circ f$ is invertible, and $(g \circ f)^{-1}$ is invertible, and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(c) The interval $(7, 9)$ is countable.

(d) If a is a non zero real numbers, then $\gcd(a, a) = a$ and $gcd(-a, a) = 1$.

(e) Let n be a positive integer greater than 1. If all prime numbers less than $\sqrt{n}$ don't divide n, then n is prime.

(f) If A is a matrix with a repeated column, then $\det(A) = 0$.

(g) If A is an $n \times n$ matrix and x and b are $n \times 1$ vectors, then the system $Ax = b$ has a unique solution.

(h) If A and B are matrices such that $AB = 0$, then either $A = 0$ or $B = 0$.

(i) $\text{lcm}(a, b) = \text{lcm}(b, -a)$.

(j) If a and b are vectors such that $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

(k) If f is a function from A to B, $X \subseteq A$, and $Y \subseteq B$, then the image of the preimage of Y is equal to Y, and the preimage of the image of X is equal to X.

(l) The union of a countable set and an uncountable set is uncountable.

(a) (8 points) Find $\gcd(100, 41)$ and write it as a linear combination of 100 and 41. Find also $\mathrm{lcm}(41, -100)$.

(b) (3 points) Find the canonical factorization of 20000.

(c) (3 points) Find 41^{-1} in $\mathbb{Z}_{100}$.

(d) (3 points) Find $200 \oplus 50$ in $\mathbb{Z}_{100}$.

(e) (3 points) Solve the equation $400x = 1$ in $\mathbb{Z}_{10000}$.

(f) (3 points) Solve the equation $4x = 0$ in $\mathbb{Z}_{10}$.

(8) Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 5 & 4 \\ 3 & 2 & 0 \end{bmatrix}.$$

(a) (7 points) Find $(2A - 3I)^T$.

(b) (8 points) Find A^2.

(c) (6 points) Find $\det(A)$.

(d) (8 points) Find A^{-1}.

(e) (4 points) Use A^{-1} to find the solution of the system (do not use substitution or elimination):
$x_1 + 2x_2 + 3x_3 = -1.$
$4x_1 + 5x_2 + 7x_3 = -2.$
$3x_1 + 6x_2 = -3.$