Assignment 4

Due Friday, March 20, 07 at 11:00 AM in Class

Remarks: I may not grade all assignments and may not grade all questions/parts on the assignment I decide to grade. You're welcome to ask me for help. Show your work and explain every step.

(1) Let $f: \mathbb{R}-\{2\} \longrightarrow \mathbb{R}^+$, $f(x) = e^{(\frac{3x-7}{x-2})}$.
   (a) Is f well-defined? Explain.
   (b) If the answer to the first part is yes, is f onto? Explain.
   (c) If the answer to the first part is yes, is f one-to-one? Explain.

(2) Prove the following functions are bijections by proving they are one-to-one and onto. Also, find the inverse of each function:
   (a) $f: (3,\infty) \longrightarrow \mathbb{R}$, $f(x) = 2 + \frac{1}{2}\ln(x-3)$.
   (b) $f: \mathbb{R}-\{2\} \longrightarrow \mathbb{R}-\{3\}$, $f(x) = \frac{3x-7}{x-2}$.

(3) Prove by counterexamples that the following functions are not one-to-one and not onto:
   (a) $f: \mathbb{R} \longrightarrow (-66.5,\infty)$, $f(x) = 9(x-777)^8 - 66$.
   (b) $f: \{1,2,3,...,n\} \longrightarrow \mathbb{Z}$, $f(k) = \frac{n!}{k!(n-k)!}$, where n is an integer greater than 10. (Do not substitute any value for n. Thus, in your counterexample, the two values in the domain that have the same image must be in terms of n.)

(4) Decide if $g \circ f$ is defined in each of the folklowing cases. If it's defined, find it:
   (a) $f: \mathbb{R} \longrightarrow [3,\infty)$, $f(x) = 2x^2 + 3$, $g: \mathbb{R} \longrightarrow \mathbb{R}$, $g(x) = 4x - 9$.
   (b) $g: \mathbb{R} \longrightarrow [3,\infty)$, $g(x) = 2x^2 + 3$, $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 4x - 9$.
   (c) $g: \mathbb{R}^+ \longrightarrow \mathbb{R}$, $g(x) = \ln(x)$, $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 4x - 9$.

(5) Prove that $|5\mathbb{Z}+1| = |10\mathbb{Z}+8|$ by finding a bijection between the two sets.