

Assignment 3

Due Wednesday, March 7, 06 at 11:00 AM in Class

Remarks: I may not grade all assignments, and may not grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit. You should do the assignment by yourself.

- (1) Determine if each of the following binary relations is reflexive, symmetric, antisymmetric, transitive, equivalence relation, partial order:

- (a) Let R be the relation on $\mathbb{Z}$ defined by

$$a R b \text{ if and only if } b - 5 < 2a < b + 5.$$

- (b) Let R be the relation on $\mathbb{Z}$ defined by

$$a R b \text{ if and only if } a + b \text{ is even.}$$

- (2) Define the following relation on $\mathbb{R}^2$:

$$(a, b)R(c, d) \text{ iff } a^2 + b^2 = c^2 + d^2.$$

Find $[(\alpha, \beta)]$. What does $[(\alpha, \beta)]$ represent in the Cartesian plane?

- (3) Let U be the unit disc in $\mathbb{R}^2$, $X = \{x + \frac{1}{5} \mid x \in \mathbb{Z}\}$ and let $Y = \mathbb{Z} \cup \{\frac{1}{5}\}$. Also, let $G = \{(x, y) \in X \times X \mid |x| \leq \frac{4}{7}\}$, $A = U \cap (Y \times Y)$, $B = U \cap G$, and $C = U \cap (\mathbb{Z} \times \mathbb{Z})$.

- (a) List the elements of A .

- (b) List the elements of B .

- (c) Find $A \triangle C$.

- (d) Find $B \triangle C$.

- (4) Let

$$A = \{(-1, 2), (4, 5), (0, 0), (6, -5), (5, 1), (4, 3)\}$$

$$B = \{b \mid b = k^2 \text{ for some } k \in \mathbb{Z} \text{ and } (a, b) \in A \text{ for some } a\}.$$

$$C = \{x - 4 \mid x \in \mathbb{Z} \text{ and } \frac{x^2 - 17x + 70}{-1576} \geq 0\}.$$

Find $(C \cup B) \cap \{\{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{3\}, -2, -1, \phi\}$.

(5) Let $\{u_n\}_{n=1}^{\infty}$ be defined by

$$u_n = \frac{2}{3} - \left(\frac{1}{6}\right)\left(\frac{1}{4^{n-2}}\right), \forall n \in \mathbb{N}.$$

Let $A = (-5, \frac{11}{3}] \cap [\frac{7}{4}, 5) \cap \mathbb{Z}^+$ and let $S = A - \{-4, 7\}$. Find

$$\sum_{i \in S} u_i$$