

Assignment 1

Due Wed, Feb 14, 06 at 11:00 AM in Class

Remarks: I may not grade all assignments, and may not grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit. You should do the assignment by yourself.

- (1) Express the following statements as formulas and find the negation of the formula:
 - (a) There exists an integer x such that for every integer y , $x + y = 7$.
 - (b) For every integer y , there exists an integer x such that $x + y = 7$.
 - (c) n is an odd integer if and only if n^2 is an odd integer.
- (2) Decide if the following statements are true or false. Explain.
 - (a) $(x + 1)^2 \geq x^2$, $\forall x \in \mathbb{R}$.
 - (b) $\forall x, y \in \mathbb{R}$, $2x^2 + 3y^2 > 0$.
 - (c) $\exists x, y \in \mathbb{R}$, $2x^2 + 3y^2 > 0$.
 - (d) There exists an integer x such that for every integer y , $x + y = 7$.
 - (e) For every integer y , there exists an integer x such that $x + y = 7$.
- (3) Express the following as a formula and find its negation, its converse, and its contrapositive.

If x is rational and y is irrational, then $x + y$ is irrational.

- (4) Determine if $(p \wedge \bar{q}) \vee (\bar{p} \wedge q)$ is logically equivalent to $(\bar{p} \vee \bar{q})$. Explain.
- (5) Prove or disprove the following (i.e. if the statement is true, prove it, and if the statement is false, write down a counterexample).
 - (a) The set $S = \{\frac{1}{2^n} \mid n \in \mathbb{N}\}$ has no smallest element.
 - (b) $2x^2 - 8x + 15 > 5$, $\forall x \in \mathbb{R}$.
 - (c) If n is a prime number, then $n^2 + n + 41$ is prime.
 - (d) If x is irrational, then $\frac{x}{2}$ is irrational.
 - (e) For every integer n , $n^2 + n$ is even.

- (f) If a and d are real numbers such that $a < d$, then there exist real numbers b and c , such that $a < b < c < d$.