CSIT 241 - Spring 03 - Exercises

1. Find the inverse of $f:\ \mathbb{R} \longrightarrow (4,\infty)$, $f(x) = 4 + e^{3x-5}$ if it exists.

   Solution: The function is a bijection (a similar example was done in class). Hence, it's invertible. Now let's find the inverse. First, replace $f(x)$ by y.

   $y = 4 + e^{3x-5}$.

   $y - 4 = e^{3x-5}$.

   $ln(y-4) = ln(e^{3x-5})$.

   $3x - 5 = ln(y-4)$.

   $3x = 5 + ln(y-4)$.

   $x = \frac{5+ln(y-4)}{3}$.

   Hence, $f^{-1}(x) = \frac{5+ln(x-4)}{3}$. Note that f^{-1} is a bijection from $(4,\infty)$ to $\mathbb{R}$. Note also $|\mathbb{R}| = |(4,\infty)|$, and hence, $(4,\infty)$ is uncountable.

2. Find the inverse of $f:\ (\frac{5}{3},\infty) \longrightarrow \mathbb{R}$, $f(x) = 4 + ln(3x-5)$ if it exists.

   Solution: The function is a bijection (a similar example was done in class). Hence, it's invertible. Now let's find the inverse. First, replace $f(x)$ by y.

   $y = 4 + ln(3x-5)$.

   $y - 4 = ln(3x-5)$.

   $e^{y-4} = e^{ln(3x-5)}$.

   $3x - 5 = e^{y-4}$.

   $3x = 5 + e^{y-4}$.

   $x = \frac{5+e^{y-4}}{3}$.

   Hence, $f^{-1}(x) = \frac{5+e^{x-4}}{3}$. Note that f^{-1} is a bijection from $\mathbb{R}$ to $(\frac{5}{3},\infty)$. Note also $|\mathbb{R}| = (\frac{5}{3},\infty)$, and hence, $(\frac{5}{3},\infty)$ is uncountable.

3. Find the inverse of $f:\ \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 4 + (3x-5)^7$ if it exists.

   Solution: The function is a bijection (a similar example was done in class). Hence, it's invertible. Now let's find the inverse. First, replace $f(x)$ by y.

   $y = 4 + (3x-5)^7$.

   $y - 4 = (3x-5)^7$.

   $(y-4)^{\frac{1}{7}} = 3x - 5$.

   $3x = 5 + (y-4)^{\frac{1}{7}}$.

   $x = \frac{5+(y-4)^{\frac{1}{7}}}{3}$.

   Hence, $f^{-1}(x) = \frac{5+(x-4)^{\frac{1}{7}}}{3}$. Note that f^{-1} is a bijection from $\mathbb{R}$ to $\mathbb{R}$.