Assignment 6

Due Wednesday, April 19, 06, at 10:00 AM in class

Instructions: Show your work and explain every step. You may not be given credit at all for incomplete solutions. Answers with no explanations will not be accepted. Do not write numbers in the decimal form (keep them as fractions). Note that I may not grade all assignments, and I may grade only selected question(s) and only selected parts of the selected questions of the assignments I choose to grade. Work on the assignment alone. But, you're welcome to ask me for help. Use only the notation used in class.

(1) Prove the following two functions are bijections and find their inverses:
   (a) $f:\ \mathbb{R}-\{\frac{4}{7}\} \longrightarrow \mathbb{R}-\{\frac{5}{7}\},\ f(x)=\frac{5x}{7x-4}$.
   (b) $f:\ \mathbb{R}-(5,\infty),\ f(x)=5+6e^{7x+8}$.

(2) Let $f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x)=3(x-4)^2+5$.
   (a) Prove by a counterexample that f is not one-to-one.
   (b) Prove by a counterexample that f is not onto.

(3) (a) Prove that $4\mathbb{N}+5$ is countable by finding a bijection from $\mathbb{N}$ to $4\mathbb{N}+5$.
   (b) Prove that $|(5,\infty)|=\mathbb{R}$ by finding a bijection from $\mathbb{R}$ to $(5,\infty)$. Note that proves $(5,\infty)$ is uncountable.

(4) Let $f:\ \mathbb{R} \longrightarrow \mathbb{R}^+,\ f(x)=3e^{2x-1}$ and $g:\ \mathbb{R}^+ \longrightarrow (0,1),\ g(x)=\frac{1}{x+1}$. Find $g\circ f$ if it's defined and $f\circ g$ if it's defined.