Assignment 5

Due Monday, April 3, 05, at 10:00 AM in class

Instructions: Show your work and explain every step. You may not be given credit at all for incomplete solutions. Answers with no explanations will not be accepted. Do not write numbers in the decimal form (keep them as fractions). Note that I may not grade all assignments, and I may grade only selected question(s) and only selected parts of the selected questions of the assignments I choose to grade. Work on the assignment alone. But, you're welcome to ask me for help. Use only the notation used in class.

(1) Decide if each of the following binary relations is reflexive, symmetric, anti-symmetric, transitive, partial order, total order, equivalence relation.

(a) $A = \mathbb{Z}$, $\mathcal{R} = \{(x, y) \in A \times A \mid x - y \text{ is odd})\}$.

(b) $A = \mathbb{R}$, $\mathcal{R} = \{(x, y) \in A \times A \mid x - y > 8\}$.

Solution:

- Not reflexive, because, for example, $(1, 1) \notin R$ since $1 - 1 < 8$.
- Since R is not reflexive, it is not an equivalence relation, not a partial order, and not a total order.
- It is not symmetric because, for example, $(10, 1) \in R$ (because $10 - 1 > 8$), but $(1, 10) \notin R$ (because $1 - 10 < 8$).
- It's antisymmetric, because if (x, y) and (y, x) are both in R, then $x - y > 8$ and $y - x > 8$. Add the two inequalities to get $(x - y) + (y - x) > 8 + 8$, which implies $0 > 16$. This is impossible. So, we can't have both (x, y) and (y, x) in R.
- It's transitive because if (x, y) and (y, z) are in R, then $x - y > 8$ and $y - z > 8$. Add the two inequalities to get $(x - y) + (y - z) > 8 + 8$. Thus, $x - z > 16$, which implies $x - z > 8$. Thus, $(x, z) \in R$.

(2) Let $\mathcal{R}_1 = \{(2, d), (4, y), (6, d), (8, z)\}$ and $\mathcal{R}_2 = \{(g, 2), (w, 3), (d, 3), (d, 5), (y, 7)\}$. Find $\mathcal{R}_2 \circ \mathcal{R}_1$.

(3) Let $A = \{1, 2, 3, 4, 6\}$ and $\mathcal{R} = \{(x, y) \in A \times A \mid x \text{ is a multiple of } y\}$. Find the digraph representation of $\mathcal{R}$ and the matrix representation of $\mathcal{R}$.

(4) Let $A = \mathbb{R}$ and $\mathcal{R} = \{(x, y) \in A \times A \mid x^2 - 1 \geq 3y\}$. Find $\mathcal{R}^{-1}$.

(5) Let $A = \mathbb{Z}$ and $\mathcal{R} = \{(x, y) \in A \times A \mid x - y \text{ is a multiple of } 3\}$. Find $[2]$.