

## Assignment 3

*Due Friday, Feb 24, 06 at 10:00 AM in Class*

**Remarks:** I may not grade all assignments, and may NOT grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit.

- (1) Find the quotient and the remainder when  $b$  is divided by  $a$ , where
  - (a)  $b = 1000$ ,  $a = 99$ .
  - (b)  $b = 1000$ ,  $a = -99$ .
  - (c)  $b = -1000$ ,  $a = 99$ .
  - (d)  $b = -1000$ ,  $a = -99$ .
- (2) Find  $\gcd(a, b)$ , find integers  $\alpha$  and  $\beta$  such that  $\gcd(a, b) = \alpha a + \beta b$ , and find  $\text{lcm}(a, b)$ , where  $a = 1000$  and  $b = 99$ .
- (3) Using the result of the previous question, find  $\gcd(b, -a)$  and find two integers  $m$  and  $n$  such that  $mb + n(-a) = \gcd(b, -a)$ , where  $a$  and  $b$  are as in the previous question.
- (4) Find the following in  $\mathbb{Z}_{1000}$ 
  - (a)  $900 \oplus 500$ .
  - (b)  $3 \odot 900$ .
  - (c)  $-99$ .
  - (d)  $99^{-1}$ .
- (5) Find the canonical factorization of 14553.