Assignment 2

Due Wednesday, Feb 15, 06 at 10:00 AM in Class

Remarks: I may not grade all assignments, and may NOT grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit.

(1) Prove or disprove the following (i.e. if the statement is true, prove it, and if the statement is false, write down a counterexample).
   (a) There is no smallest negative integer.
   (b) $2x^2 - 8x + 15 > 5$, $\forall x \in \mathbb{R}$.
   (c) If n is prime, then n^2 is prime.
   (d) If x is irrational, then $\frac{x}{5}$ is irrational.
   (e) For every integer n, $n^2 + n$ is even.
   (f) If a and f are real numbers such that $a < f$, then there exist real numbers b, c, d, and e, such that $a < b < c < d < e < f$.

(2) Prove the following by mathematical induction:
   (a) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$, $\forall n \geq 1$.

   (b) $n! \geq 2^{n-1}$, $\forall n \in \mathbb{Z}^+$.