Final Exam

(1) Find the point (x-coordinate only) of intersection in the first quadrant between $g(x) = e^x$ and h(x) $= x^2$ using Newton's method. The termination conditions are:
   (a) $|ftol| < 0.00001$.
   (b) The maximum number of iterations is 4.

(2) Approximate the solution of the following system by the Jacobi method. Find only the first and the second iterations. Start with $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$$10x_1 - 2x_2 + x_3 = 9.$$

$$-2x_1 + 10x_2 - 2x_3 = 12.$$

$$-2x_1 - 5x_2 + 10x_3 = 18.$$

(3) Find Newton's interpolating polynomial p(x) of degree 3 that interpolates function f at the points (2,3), (3,2), (5,1.5), (9,1.25).

(4) Find the first 4 approximations (i.e. Find $T(k),\ k = 0, \cdots, 3$) of $\int_0^1 e^{x^2}\ dx$ using the recursive Trapezoidal rule.

(5) Approximate the solution of the IVP

$$y' = xy,\ x \in [0, 2], y(0) = 1,$$

using the Midpoint method with $n = 2$.

(6) Find the LU factorization of

$$\begin{bmatrix} 3 & 0 & 1 \\ 0 & -1 & 3 \\ 1 & 3 & 0 \end{bmatrix}$$

(7) (a) Let p_n be Newton's interpolating polynomial of degree n of $f(x)$ at $x_0, x_1, x_2, \cdots, x_n$. Find the coefficient of x^{n-1} in p_n.

(b) Let $f(x) = \frac{1}{x}$, and let $x_0, x_1, x_2, \cdots, x_n$ be nonzeros. Prove that

$$f[x_0, x_1, x_2, \cdots, x_n] = (-1)^n \prod_{i=0}^{n} \frac{1}{x_i}.$$

———————————- End of the Final Exam ———————————-