

## Facts about Hermitian, Unitary, Skew-hermitian, and Normal Matrices, and Others

- (1) Every matrix  $M$  can be expressed as the sum of a Hermitian matrix and a skew-Hermitian matrix. I.e., we can write  $M = \frac{1}{2}(M + M^H) + \frac{1}{2}(M - M^H)$ .
- (2) A skew-Hermitian matrix is also called antiHermitian.
- (3) A real matrix is skew-Hermitian iff it's skew-symmetric.
- (4) The real part of every element of the main diagonal of a skew-Hermitian matrix has a zero real part.
- (5) Every skew-Hermitian matrix is normal.
- (6)  $A + Bi$  ( $A$  and  $B$  are real matrices) is skew-Hermitian iff  $A$  is skew-symmetric and  $B$  is symmetric.
- (7) A real matrix is Hermitian iff it's symmetric.
- (8) The main diagonal of a Hermitian matrix is real.
- (9) The eigenvalues of a Hermitian matrix are real (but the eigenvectors can be complex).
- (10) Eigenvectors of a Hermitian matrix that correspond to different eigenvalues are orthogonal.
- (11) Every Hermitian matrix is normal.
- (12)  $A + Bi$  ( $A$  and  $B$  are real matrices) is Hermitian iff  $A$  is symmetric and  $B$  is skew-symmetric.
- (13) Every unitary matrix is normal.
- (14) If  $M$  is unitary, then  $\|Mx\|_2 = \|x\|_2, \forall x \in \mathbb{C}^n$ .
- (15) If  $\lambda$  is an eigenvalue of a unitary matrix, then  $|\lambda| = 1$ .
- (16) The columns of a unitary matrix are pairwise orthogonal and the two-norm of each column is 1.
- (17) An  $n \times n$  matrix is normal iff it has  $n$  orthogonal eigenvectors.