

Questions about Permutations, Combinations, and Derangements

1. Evaluate $\sum_{k=0}^n (-1)^k C(n, k) 3^{n-k}$.
2. Use the Binomial Theorem to simplify 1.01^3 .
3. Prove each of the following with no more than one line
 - $\sum_{k=0}^n \frac{1}{k+1} 2^{k+1} C(n, k) = \frac{3^{n+1}}{n+1}$.
Solution: Consider $(x+1)^n = \sum_{k=0}^n C(n, k) x^k$. Now take the integral of this equation with respect to x and then substitute in the result $x = 2$.
 - $\sum_{k=1}^n k C(n, k) = n 2^{n-1}$. You have to use here a method different than that we used in class. Remember this was solved in class but by a different method.
Solution: Consider $(x+1)^n = \sum_{k=0}^n C(n, k) x^k$. Now take the derivative of this equation with respect to x and then substitute in the result $x = 1$.
4. With no more than one line, evaluate $\sum_{k=1}^n k^2 C(n, k)$.
Solution: Consider $(x+1)^n = \sum_{k=0}^n C(n, k) x^k$. Now take the derivative of this equation with respect to x twice and then substitute in the result $x = 1$.
5. You are given 33 points in the cartesian plane. No 3 of them are collinear. How many triangles can you make of these points if the vertices of each triangle have to be from the given points.
Solution: $C(33, 3)$.
6. 23 persons are to sit around a circular table. This group of people includes A and B who don't like each other and don't want to sit next to each other. In how many ways can the 23 persons sit.
Solution: $(20)(21!)$. (Get it from $22! - (2! \cdot 21!)$)
7. Prove that $D_n = (-1)^n + n D_{n-1}$.
8. Use the above (what you've just proved) to prove in no more than two lines that D_n is even if n is an odd positive integer and odd if n is an even positive integer.