

CSIT 241 - Determinants

NAME:

1. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. Then $\det(A)$ (also denoted $|\det(A)|$) is equal to $a_{11}a_{22} - a_{12}a_{21}$.

2. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Then $\det(A) = a_{11} \cdot \det(A_1) - a_{12} \cdot \det(A_2) + a_{13} \cdot \det(A_3)$, where $A_1 = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$, $A_2 = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$, and $A_3 = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$.

To find the determinants of A_1 , A_2 , and A_3 , you use the previous part. Note also we got A_1 by omitting the row and the column of a_{11} , we got A_2 by omitting the row and the column of a_{12} , and we got A_3 by omitting the row and the column of a_{13} . Note also in the formula for the determinant we alternate between + and - starting with +.

3. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$. Then $\det(A) = a_{11} \cdot \det(A_1) - a_{12} \cdot \det(A_2) +$

$$a_{13} \cdot \det(A_3) - a_{14} \cdot \det(A_4), \text{ where } A_1 = \begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}, A_2 = \begin{bmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{bmatrix}, \text{ and } A_4 = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}.$$

To find the determinants of A_1 , A_2 , A_3 , and A_4 , you use the previous part. Note also we got A_1 by omitting the row and the column of a_{11} , we got A_2 by omitting the row and the column of a_{12} , we got A_3 by omitting the row and the column of a_{13} , and we got A_4 by omitting the row and the column of a_{14} . Note also in the formula for the determinant we alternate between + and - starting with +.

4. Look at the pattern and you'll be able to generalize the procedure described above (for finding the determinant), to any $n \times n$ matrix.
5. Let A and B be $n \times n$ matrices, let $r \in \mathbb{R}$, and let $k \in \mathbb{Z}$. Then

(a) $\det(AB) = \det(A) \cdot \det(B)$.

(b) $\det(A^k) = (\det(A))^k$.

- (c) $\det(A^T) = \det(A)$.
- (d) If A is invertible, then $\det(A^{-1}) = \frac{1}{\det(A)}$.
- (e) In general $\det(A + B) \neq \det(A) + \det(B)$.
- (f) If A is diagonal or upper triangular or lower triangular, then the determinant of A is equal to the product of the elements of the main diagonal of A . In particular, $\det(Z) = 0$ and $\det(I) = 1$.
- (g) $\det(rA) = r^n \cdot \det(A)$.
- (h) If A has a zero row or a zero column, then $\det(A) = 0$.
- (i) Interchanging two rows of A results in multiplying the determinant by -1.
- (j) Adding a multiple of one row to another does not change the determinant.
- (k) Multiplying a row by a number α results in multiplying the determinant by α .