

Quiz 4

Name:

Instructions: Show your work and explain every step.

- (1) (4 points) Find the inverse of $f : \mathbb{R} \longrightarrow (4, \infty)$, $f(x) = 4 + e^{5x-6}$.

- (2) (6 points) Prove by counterexamples that the following function is not onto and not one-to-one:
 $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 2(x+7)^2 - 9$.

- (3) (4 points) Prove that $|\mathbb{R}| = |(4, \infty)|$ by finding a bijection between the two sets.

- (4) (6 points) Determine if the following are true or false:
 - (a) If $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are functions such that $g \circ f$ is a bijection, then f is onto and g is one-to-one.
 - (b) If $f : A \longrightarrow B$ is a function (not necessarily invertible), $X \subseteq A$ and $Y \subseteq B$, the preimage of set M is $f^{-1}(M)$ and the image of set N is $f(N)$, then
$$f^{-1}(f(X)) = X \text{ and } f(f^{-1}(Y)) = Y.$$
 - (c) $\mathbb{Q}^- \cup 5\mathbb{Z}$ is countable.