CSIT 241 - EXAM II

Name:

Instructions: Do all of the following. EXPLAIN every step. Points will be deducted for incomplete proofs or incomplete solutions. Do not use calculators. Do not look at your neighbor or talk to him. If you use a method different than what the question is asking for, you'll get no credit. Use only notation used in class. Any violation of the instructions may result in a partial credit or no credit at all.

The exam is closed book, closed notes. No material may be used and the Internet is not allowed.

Time: 50 minues.

(1) (10 points) Find the inverse of the function $f : \mathbb{R}^+ \longrightarrow (1, e^2)$, defined by $f(x) = e^{(\frac{2x}{x+3})}$.

(2) (12 points) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 7(x+50)^2 - 100$.

(a) Prove by a counterexample that f is not one-to-one.

(b) Prove by a counterexample that f is not onto.

(3) (7 points) Prove or disprove the following (provide full explanation):

The union of a countable set and an uncountable set is uncountable.

(4) (6 points) Determine if the following are true or false:

(a) If $f: A \longrightarrow B$ and $g: B \longrightarrow C$ are functions such that $g \circ f$ is a bijection, then g is onto and f is one-to-one.

(b) If $f: A \longrightarrow B$ is a function (not necessarily invertible), $X \subseteq A$ and $Y \subseteq B$, the preimage of set M is denoted by $f^{-1}(M)$ and the image of set N is denoted by $f(N)$, then

$$f(f^{-1}(Y)) = Y.$$

(5) (6 points) Find $\gcd(251, 100)$ and write it as a linear combination of 251 and 100.

(6) (20 points)

(a) Find 100^{-1} in $\mathbb{Z}_{251}$.

(b) Find $200 \oplus 250$ in $\mathbb{Z}_{251}$.

(c) Find -300 (mod 251).

(d) Given the equation: $(-74)(400) + (99)(299) = 1$. Find 101^{-1} in $\mathbb{Z}_{299}$.

(e) Solve the equation $650x = 1$ in $\mathbb{Z}_{40000000}$.

(7) (7 points) Solve the following system in $\mathbb{Z}_6$:

$x + y = 2.$

$2x + y = 1.$

(8) (32 points) Answer 8 and only 8 of the following. If you answer more than 8, you may get no credit, or only the first 8 will be graded.

(a) In how many ways can the letters of the English alphabet be arranged so that there are 13 letters between b and c? **Answer:**

(b) How many six-digit decimal numbers are palindromes? **Answer:**

(c) How many permutations of the word PALINDROMES contain a permutation of ROME? **Answer:**

(d) How many numbers in the set $\{1, 2, 3, \cdots, 2000\}$ are divisible by 7 but not by 6? **Answer:**

(e) In how many ways can you arrange 20 distinct CS books, 30 distinct Math books, and 40 distinct Biology books on a shelf if the CS books are to be together and the Math books are to be together? **Answer:**

(f) In how many ways can you reorder the letters of the word

PALINDROMES so that exactly 4 of them remain in their original positions? Write the answer in terms of derangements. **Answer:**

(g) In how many ways can you reorder the letters of the word PALINDROMES so that at least one of them remains in its original position? Write the answer in terms of derangements.
Answer:

(h) How large a group of people coming from 50 different countries should be to ensure 30 come from the same country? **Answer:**

(i) In how many ways can you distribute 30 (distinct) red balls and 50 (identical) blue balls into 200 distinct boxes at most one ball to a box?
Answer:

(j) In how many ways can you distribute 30 (distinct) red balls and 50 (identical) blue balls into 200 distinct boxes? (Here there is no limit on the number of balls in each box.) **Answer:**

(k) In how many ways can you order the letters of the word JUSTIFIABILITY so that no two I's are consecutive? **Answer:**