CSIT 241 - EXAM I

Name:

Instructions: Do all of the following. EXPLAIN every step. Points will be deducted for incomplete proofs or incomplete solutions. Do each question on a separate sheet of paper, write your name on each paper, and staple them. Do not use calculators. Do not look at your neighbor or talk to him. If you use a method different than what the question is asking for, you'll get no credit. Use only notation used in class. Any violation of the instructions may result in a partial credit or no credit at all.

Other instructions: The exam is closed book, closed notes. No material can be used and the Internet is not allowed.

Time: 50 minues.

(1) (15 points)

(a) Let $B = \{(2,3),(2,2),(2,1),(1,2),(4,2),(-1,1),(1,-1),(1,0)\}$, and $A = \{\frac{x}{y} \in \mathbb{Z} \mid (x,y) \in B\}$. List the elements of A.

(b) Let $A = (2,7]$ and $B = (4,8]$. Find $A - B$.

(c) Let $A = [3,9)$. Find $A \cap 3\mathbb{Z}$.

(d) Let $A = \{b,c\}$. Find $\mathcal{P}(A)$.

(e) Let A be the interval $(1,3)$ and B be the interval $[2,4]$. Draw $A \times B$ in the Cartesian plane.

(2) (6 points) Find the negation and the contrapositive (indicate which is which) of the following:

If x is greater than 2 and y is greater than 3, then xy is greater than 6.

(3) (6 points) Determine if the following is true or false and find its negation:

There exists an integer x such that for every even integer y, $y = 2x$.

(4) (10 points) Prove the following WITHOUT using truth tables

$$(p \wedge q) \longrightarrow r \equiv p \longrightarrow (\overline{q} \vee r).$$

(5) (18 points) The following statements are true. Prove them.

(a) The set $\{\frac{-1}{n} \mid n \in \mathbb{N}\}$ has no largest element.

(b) $x^2 + 6x + 15 > 4$, $\forall x \in \mathbb{R}$.

(c) If a and d are real numbers with $a < d$, then there exist real numbers b and c such that $a < b < c < d$.

(6) (8 points) The following statements are false. Give counterexamples to prove they are false.

(a) If n is prime, then $n^2 + n + 41$ is prime.

(b) If a and b are irrational, then $a + b$ is irrational.

(7) (10 points) Prove the following by mathematical induction.

$$1 + 3 + 3^2 + \cdots + 3^{n-1} = \tfrac{3^n - 1}{2}, \ \forall n \geq 1.$$

(8) (10 points) Prove the following by resolution

(1) $(p \longrightarrow q) \vee r$.

(2) $p \vee s$

(3) $\overline{q \vee r}$

Conclusion: s.

(9) (16 points)

(a) Let $A = \mathbb{Z}$, $\mathcal{R} = \{(x, y) \in A \times A \mid x - y > 5)\}$. Prove by a counterexample that $\mathcal{R}$ is not symmetric.

(b) Let $A = \mathbb{R}$, $\mathcal{R} = \{(x, y) \in A \times A \mid x - y > 5\}$. Prove that $\mathcal{R}$ is transitive.

(c) Let $A = \mathbb{Z} - \{0\}$, $\mathcal{R} = \{(x, y) \in A \times A \mid x \text{ divides } y\}$. Prove by a counterexample that $\mathcal{R}$ is not antisymmetric.

(d) Let Let $A = \mathbb{Z}^+$, $\mathcal{R} = \{(x, y) \in A \times A \mid x \text{ divides } y\}$. Show by a counterexample that $\mathcal{R}$ is not a total order.

(e) Let $A = \{d, e, f\}$. Give an example of a binary relation on A that is both an equivalence relation and a partial order. You must list all the elements of the binary relation (i.e. don't write it as a formula).

(f) Can a binary relation on $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$ have (among others) the following equivalence classes: $\{2, 3, 4\}$, $\{4, 5, 6\}$? Explain.

(g) Let $\mathcal{R}$ be a an equivalence relation on set A and let 3 and 7 be elements in A. If the equivalence class of 3 is $\{3, 7, 10, 12\}$. What is the equivalence class of 7?

(h) Let $A = \mathbb{Z} - \{0\}$, $\mathcal{R} = \{(x, y) \in A \times A \mid xy > 0\}$. Find $[-5]$.