Assignment 5

Due Monday, Oct 23, 06 at 1:00 AM in Class

Remarks: I may not grade all assignments, and NOT all questions/parts will be graded on the graded assignment. You're welcome to ask me for help. Show your work and explain every step.

(1) Prove the following functions are bijections by proving they are one-to-one and onto. Also, find the inverse of each function:
 (a) $f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 4x - 9.$
 (b) $f:\ \mathbb{R} \longrightarrow (3, \infty),\ f(x) = 3 + e^{2x-4}.$
 (c) $f:\ \mathbb{R} - \{2\} \longrightarrow \mathbb{R} - \{3\},\ f(x) = \frac{3x-7}{x-2}.$

(2) Prove by counterexamples that the following functions are not onto:
 (a) $f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 3 + e^{2x-4}.$
 (b) $f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 2(x-5)^2 - 8x + 11.$

(3) Prove by counterexamples that the following functions are not one-to-one:
 (a) $f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 2(x+5)^6 - 8x + 11.$
 (b) $f:\ \{1, 2, 3, ..., n\} \longrightarrow \mathbb{N},\ f(k) = \frac{n!}{k!(n-k)!}$, where n is an integer greater than 1 (do not substitute any value for n; thus, in your counterexample, the two values in the domain that have the same image must be in terms of n).

(4) Decide if $g \circ f$ is defined in each of the folklowing cases. If it's defined, find it:
 (a) $f:\ \mathbb{R} \longrightarrow [3, \infty),\ f(x) = 2x^2 + 3,\ g:\ \mathbb{R} \longrightarrow \mathbb{R},\ g(x) = 4x - 9.$
 (b) $g:\ \mathbb{R} \longrightarrow [3, \infty),\ g(x) = 2x^2 + 3,\ f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 4x - 9.$
 (c) $g:\ \mathbb{R}^+ \longrightarrow \mathbb{R},\ g(x) = ln(x),\ f:\ \mathbb{R} \longrightarrow \mathbb{R},\ f(x) = 4x - 9.$

(5) (a) Prove that $|(2, 5)| = |(1, 9)|$ by finding a bijection between the two sets.
 (b) Prove that $S = 4\mathbb{Z} \cup \{0, 1, 2\}$ is countable by finding a bijection from N to S.

Exercises on Binary Relations

(1) Let R be the relation on $\mathbb{Z}$ defined by

$$a\ R\ b \text{ if and only if } a = |b|^3.$$

Is R antisymmetric? Is is transitive? If the answer to any of them is yes, prove that. If the answer is not, then write down a counter example. Also, find R^{-1}.

(2) Let R be the relation on $\mathbb{Z}$ defined by

$$a\ R\ b \text{ if and only if } 3 - b \leq a - 2b < 1 + b.$$

Give a counter example to show that R is not transitive. Is R symmetric? If yes, prove it. If not write down a counterexample. Think (but do not hand in) about how to find R^{-1}.