

Assignment 4

Due Tuesday at 1:00 PM in Class

Remarks: I may not grade all assignments, and may NOT grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit. **Do NOT use truth tables in any of the questions below.**

- (1) Determine whether the following binary relation $\mathcal{R}$ defined on the set $\mathbb{Z}$, is a partial order or an equivalence relation. If it's an equivalence relation, find all distinct equivalence classes. Explain.

$$\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x - y \text{ is divisible by } 4\}.$$

- (2) Determine whether the following binary relation $\mathcal{R}$ defined on the set $\mathbb{R}$, is reflexive, symmetric, antisymmetric, transitive. Explain.

$$\mathcal{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 2x + 2y > 3y + x + 2\}.$$

- (3) Let $B = \{5, 4\}$, $C = [1, 4]$, $D = (2, 6]$, $E = \{4, a\}$.

- (a) Find $\mathcal{P}(B)$.
- (b) Find $B \times E$.
- (c) Find $C \cup D$.
- (d) Find $D - C$.
- (e) Find $D \cap 2\mathbb{Z}$.
- (f) Draw (in the Cartesian plane) $C \times D$.

- (4) Let $\{u_n\}_{n=1}^{\infty}$ be defined by

$$u_n = n^2 - 2n + 1.$$

Let $A = (-5, \frac{11}{3}] \cap [\frac{7}{4}, 5) \cap \mathbb{Z}^+$ and let $S = A - \{-4, 7\}$. Find

$$\sum_{i \in S} u_i.$$

- (5) Let

$$\begin{aligned} A &= \{(-1, 2), (4, 5), (0, 0), (6, -5), (5, 1), (4, 3), (7, 9)\} \\ B &= \{b \mid b = k^2 \text{ for some } k \in \mathbb{Z} \text{ and } (a, b) \in A \text{ for some } a\}. \\ C &= \{x - 4 \mid x \in \mathbb{Z} \text{ and } \frac{-157}{x^2 - 6x + 8} \geq 0\}. \end{aligned}$$

Find $(C \cup B) \cap \{\{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{0\}, -2, -1, \phi\}$.

- (6) Let $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$, $B = \{(x, y) \in \mathbb{R}^2 \mid |x| < 1, |y| < 1\}$. Let $M = \mathbb{Z} \cup \{\frac{1}{2}\}$ and $N = \{x \mid x = \frac{1}{2} + q, q \in \mathbb{Z}\}$. Finally, let $C = A \cap (M \times M)$, $D = B \cap (M \times M)$, $E = A \cap (N \times N)$, $F = B \cap (N \times N)$. Find

- (a) $D - C$.
- (b) $F \cap D$.

- (7) Let b be the sequence defined by $b_n = 2^{n-3}$, $\forall n \in \mathbb{N}$, and let $S = \{5, 8\}$. Find $\sum_{i \in S} b_i$.

- (8) Let $B = \{(2, 3), (2, 2), (2, 1), (1, 2), (4, 2), (-1, 1), (1, -1), (1, 0)\}$, and $A = \{\frac{x}{y} \in \mathbb{Z} \mid (x, y) \in B\}$. List the elements of A .