Assignment 1

Due Wed, Sep 13, 06 at 1:00 PM in Class

Remarks: I may not grade all assignments, and may NOT grade all questions/parts on the assignments I choose to grade. You're welcome to ask me for help. Show your work and explain every step. If you don't provide enough explanation, you may get no credit or partial credit.

(1) Find the negation of the following:
   (a) There exists an integer x such that for every integer y, $x + y = 7$.
   (b) For every integer y, there exists an integer x such that $x + y = 7$.
   (c) n is an odd integer if and only if n^2 is an odd integer.

(2) Decide if the following statements are true or false. Explain.
   (a) $(x + 1)^2 \geq x^2$, $\forall x \in \mathbb{R}$.
   (b) $\forall x, y \in \mathbb{R}$, $2x^2 + 3y^2 > 0$.
   (c) $\exists x, y \in \mathbb{R}$, $2x^2 + 3y^2 > 0$.
   (d) There exists an integer x such that for every integer y, $x + y = 7$.
   (e) For every integer y, there exists an integer x such that $x + y = 7$.

(3) Find the converse, the negation, and the contrapositive of the following. Indicate which is which.

   If x is rational and y is irrational, then $x + y$ is irrational.

(4) Prove or disprove the following (i.e. if the statement is true, prove it, and if the statement is false, write down a counterexample). The set $\mathbb{R}$ is the set of real numbers and the set $3\mathbb{Z}$ is the set of all integers that are multiples of 3.
   (a) The set $3\mathbb{Z}$ has no largest element.
   (b) $2x^2 - 8x + 15 > 5$, $\forall x \in \mathbb{R}$.
   (c) If n is prime, then n^2 is prime.
   (d) If x is irrational, then $\frac{x}{5}$ is irrational.
   (e) For every integer n, $n^2 + n$ is even.
   (f) If a and f are real numbers such that $a < f$, then there exist real numbers b, c, d, and e, such that $a < b < c < d < e < f$.

(g) n is even if and only if n^2 is even.