

Quiz 1

Name:

Instructions: Show your work and explain every step. You may not be given credit at all for incomplete solutions. Answers with no explanations will not be accepted. Do not write the numbers in the decimal form (keep them as fractions).

- (1) (3 points) Find the negation, contrapositive, and the converse of the following statement (make it clear which one is which). Also, determine whether the statement is true or false.

If x is a prime number and $x > 2$, then x is odd.

- (2) (3 points) Decide whether the following statement is true or false. Also, express the statement as a mathematical formula.

There exist rational numbers x_1 and x_2 such that $x_1 < x_2$ and

$$x_1^3 - x_1 > x_2^3 - x_2.$$

- (3) (2 points) Find the negation of the following statement:

$$\forall x \in \mathbb{R} (x^2 + 1 \geq 1).$$

- (4) (5 points) Let $A = \{2, 3, 4\}$, $B = \{5, 4\}$, $C = (1, 3]$, $D = (2, 5]$.

Find the following

- (a) $\mathcal{P}(B)$.
 - (b) $A \times B$.
 - (c) $C \cup D$.
 - (d) $C - D$.
 - (e) $D \cap 2\mathbb{Z}$.
- (5) (4 points) Decide whether the following are true or false
- (a) Let A and B be sets. Then, $|A \cup B| = |A| + |B|$.
 - (b) $\overline{\overline{q} \longrightarrow \overline{p}} \equiv p \wedge \overline{q}$.
 - (c) $\{3\} \in \{1, 2, 3\}$.
 - (d) $\{3\} \subseteq \{1, 2, \{3\}\}$.
- (6) (3 points) Prove that $(p \wedge q) \vee \overline{p} \equiv p \longrightarrow q$.