CSIT 241 - Examples

1. A state wants to produce license plates consisting of 4 uppercase letters, a space, and three digits (zero is excluded). If repetition is allowed (i.e. you can repeat digits/letters), how many different license plates are possible?

   Solution: There are $(26)^4$ ways to choose 4 uppercase letters and 9^3 ways to choose 3 nonzero digits. Thus, by the multiplication principle, there are $(26)^4 \cdot 9^3$ possible different license plates.

2. An instructor has divided his class into 8 groups. Each group has to give a presentation. The instructor wants the presentations to be given in the last 3 classes of the semester. He wants 3 presentations each day except the last day in which he wants only two presentations. In how many ways can this be done?

   Solution: This is exactly as the question: In how many ways can 8 people wait in a line? Or in how many ways can we put 8 distinct balls in 8 distinct boxes at most one ball a box? So, the answer is 8! (or $P(8,8)$).

3. A systems administrator decided to make a password consist of 7 characters, the first has to be from the set $\{A, B, C, D\}$ and the remaining 6 characters can be either lowercase English alphabets or digits. How many different passwords are possible?

   Solution: We can first choose the first character. There are 4 possible ways to do that. Then choose the remaining 6 characters. There are $(26+10)^6$ possible ways to do that. Thus, by the multiplication principle, the answer is $4 \cdot (36)^6$.

4. In how many ways can you arrange the letters of the word BANANA (i.e. how many distinct permutations of the letters of BANANA are there)?

   Solution: $\frac{6!}{3! \cdot 2!}$. We have to divide by 3! because A appears 3 times and by 2! because N appears twice.

5. A committee consists of 4 women and 6 men has to be chosen from a group of 20 women and 30 men. How many different committees can be chosen?

   Solution: First choose the women. There are $C(20,4)$ ways to do that. Then choose the men. There are $C(30,6)$ ways to do that. Thus, by the multiplication principle, the answer is $C(20,4) \cdot C(30,6)$.

6. In how many ways can a person choose 4 CDs from the top ten list if repetition is allowed and if there are at least 4 CDs from each one of the top ten?

   Solution: This is the same as asking: In how many ways can you put 4 identical balls into 10 distinct boxes any number in a box? The answer is: $C(10+4-1,4)$.

7. In how many ways can the letters of the English alphabet be arranged so that there are exactly 10 letters between a and z?

   Solution: There $P(24, 10)$ ways of choosing the 10 letters between a and z. Now look at those 10 letters and at a and z as one object. Then, you have now 14+1 abjects to arrange. The number of different arrangements of them is 15!. But, also either a will be first or z will be first, so you have to multiply by 2. Thus, by the multiplication principle, the answer is: $2 \cdot P(24, 10) \cdot 15!$.

8. A man, a woman, a boy, a girl, a dog, and a cat, are walking down a road one after another. In how many ways can this happen if the dog has to be between the man and the boy?

   Solution: Think of the man, the boy, and the dog, as one object. Thus, you have 4 objects to order. You can do that in 4! different ways. Now since either the boy will be first or the man will be first, then you'll have to multiply by 2. Thus, the answer is: $2 \cdot 4!$.

9. In how many ways can 8 books be split among Jay, Mary, and Chris, if Jay has to get 4 books, and Mary and Chris each gets two books?

   Solution: The question is the same as: In how many ways can you arrange the letters of the word JJJJMMCC? So, the answer is $\frac{8!}{4! \cdot 2! \cdot 2!}$.

10. Look at all the examples we did in class. It is very important to do that.