Assignment 1

Remarks: I may not grade all assignments, and Not all questions/parts will be graded on the graded assignment. You're welcome to ask me for help. Show your work and explain every step. I'll give you some hints in class. Some of the material needed for the homework will be covered Wednesday.

(1) Prove or disprove the following.
  (a) If a and d are real numbers with $a < d$, then there exists two real numbers b and c such that $a < b < c < d$.
  (b) If a and b are real numbers with $a < b$, then $a < \frac{a+b}{2}$.
  (c) If n is a prime number greater than 2, then n is odd.
  (d) If n is a prime number greater than 2, then $n+1$ is not prime. (*Hint: see the previous item.*)
  (e) $n+1$ is odd if and only if n^2-1 is odd.
  (f) If n is a positive integer, then 2^n+1 is prime.
  (g) If n^2 is an odd integer, then n is odd.
  (h) The set $\{\frac{-1}{n} \mid n \in \mathbb{N}\}$ has a largest element.
  (i) The interval $(2,4]$ has a smallest rational element.

(2) Prove the following by mathematical induction:
  (a) $1 + a + a^2 + \cdots + a^{n-1} = \frac{a^n-1}{a-1}$, $a \neq 1$, $\forall n \in \mathbb{Z}^+$.
  (b) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$, $\forall n \geq 1$.
  (c) $1 + 3 + 3^2 + \cdots + 3^{n-1} = \frac{3^n-1}{2}$, $\forall n \geq 1$.
  (d) $(\frac{1}{2})^n \geq 1 - \frac{n}{2}$, $\forall n \geq 1$.