Homework 6

Remarks: Do not turn in this homework. These problems are for practice. We may go over some of them in class, but we will do that only after you try them at home. You have to show your work and explain every step. For elementary row operations, you must write down each operation you do.

(1) Determine whether the following are subspaces of $\mathbb{R}^2$

(a) $\{(x_1, x_2)|x_1 + x_2 = 0\}$.

(b) $\{(x_1, x_2)|x_1 x_2 = 0\}$.

(c) $\{(x_1, x_2)|x_2 = 4x_1\}$.

(d) $\{(x_1, x_2)|x_2 = 4x_1 + 3\}$.

(2) Determine whether the following are subspaces of $\mathbb{R}^3$

(a) $\{(x_1, x_2, x_3)|x_1 + x_3 = 0\}$.

(b) $\{(x_1, x_2, x_3)|x_1 = x_2 = x_3 = 0\}$.

(c) $\{(x_1, x_2, x_3)|x_3 = x_1 + x_2\}$.

(d) $\{(x_1, x_2, x_3)|x_3 = x_1^2 + x_2^2\}$.

(3) Determine whether the following are subspaces of $\P_4$

(a) The set of polynoials in P_4 of even degree.

(b) The set of polynoials of degree 3.

(c) The set of polynoials in P_4 for which 0 is a root.

(d) The set of polynoials in P_4 that have at least one real root.

(4) Let U and V be subspaces of a vector space V and let

$$W = U + V := \{u + v \mid u \in U, \ v \in V\}.$$

Show that W is a subspace of V.

(5) Let $v_1, \ v_2, \cdots, v_m$ be linearly independent vectors in $\mathbb{R}^n$ and let A be a nonsingular $n \times n$ matrix. Let $w_i = Av_i$, $i = 1, \cdots, m$. Show that w_i, $i = 1, \cdots, m$ are linearly independent.

(6) Let $x_1 = (2, 1, 3)$, $x_2 = (3, -1, 4)$, and $x_3 = (2, 6, 4)$. Show that $x_1, \ x_2, \ x3$ are linearly independent. Also, show that x_1 and x_2 are linearly independent.

(If T is linearly independent and $S \subset T$, what can you say about T? If T span V and $T \subset S$, what can you say about S?)

(7) If V is a vector space of dimension $n > 0$. Prove that
   (a) Any set of n linearly independent vectors spans V.
   (b) Any n vectors that span V are linearly independent.
   (c) If $m > n$, then any collection of m vectors in V is linearly dependent.
   (d) No set of less than n vectors can span V.
   (e) Any subset of less than n linearly independent vectors can be extended to form a basis for V.

(8) Let $S = \{v_1, \cdots, v_m\}$ be a set in a vector space V. Prove that
   (a) If one of the vectors in S, say v_j, is a linear combination of the remaining vectors in S, then span $S - \{v_j\} =$ span S.
   (b) If Span $S \neq \{0\}$. Then some subset of S is a basis for span S.

(9) Prove that P is infinite-dimensional.

(10) Page 278: 11, 12, 17, 19.

(11) Page 288: 7, 9, 11, 17, 25, 27, 33, T.6, T.9.

(12) Page 301: 1, 3, 5, 11, 12, 13.

(13) Page 314: 1, 3, 5, 6, 7, 9, 11, 13, 15, 17, 19, 21, 23, 27, 29, 31, 33.