Homework 4

Remarks: Do not turn in this homework. These problems are for practice. We may go over some of them in class, but we will do that only after you try them at home. You have to show your work and explain every step. For elementary row operations, you must write down each operation you do.

(1) Find the row echelon form of the following matrix

$$\begin{bmatrix} 0 & 2 & 3 & 4 \\ 4 & 0 & 5 & 6 \\ 1 & 2 & 4 & 3 \end{bmatrix}$$

(2) Use the previous question to solve the following system

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 3. \\ 2x_2 + 3x_3 &= 4. \\ 4x_1 + 5x_3 &= 6. \end{aligned}$$

(3) Let u and v be solutions of the homogeneous linear system $Ax = 0$.
  (a) Show that $u + v$ is a solution of the system.
  (b) If r is a number, show that ru is a solution of the system.

(4) Show that if u and v are solutions of the linear system $Ax = b$, then $u - v$ is a solution of the homogeneous system $Ax = 0$. Now assume $b \neq 0$ and assume the system $Ax = b$ is consistent. Show that if x is a particular solution of $Ax = b$ and y is a solution to $Ax = 0$, then $x + y$ is a solution of $Ax = b$.

(5) Without computing the determinant, find all values of a for which A is invertible, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$$

(6) Show that the following matrix is nonsingular and find its inverse

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

(7) **Definition** Let B and A be $n \times n$ matrices. We say B is *similar* to A if there exists a nonsingular matrix P such that $B = P^{-1}AP$.

(a) Show that the similarity relation on matrices is an equivalence relation.

(b) Let A be an $n \times n$ matrix. We define e^A to be $\sum_{k=0}^{\infty} \frac{1}{k!}A^k$. Now let B be a matrix similar to A (i.e. $B = P^{-1}AP$). Express e^B in terms of P, and P^{-1}, and powers of A. (You should not use powers of P or powers of P^{-1}.

(8) Let A be an $n \times n$ skew-symmetric matrix and let $x \in \mathbb{R}^n$ (i.e. x is an $n \times 1$ real vector). Show that $x^T Ax = 0$.