

Assignment 1

Due: Monday, Sep 27, 04, at 2:00 pm (must be turned in in class)

Instructions: Show your work and explain every step. You may not be given credit at all for incomplete solutions. Answers with no explanations will not be accepted. Do not write the numbers in the decimal form (keep them as fractions). Note that I may grade only selected question(s). Solve each problem on a different page and staple them. Also, write your name on each page.

(1) Let

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 7 \\ 3 & 4 & 5 & 6 & 3 \\ 4 & 5 & 6 & 2 & 4 \\ 5 & 6 & 2 & 3 & 1 \\ 6 & 2 & 3 & 4 & 2 \end{bmatrix}.$$

Write A as a sum of a symmetric matrix and a skew-symmetric matrix. I.e. find a symmetric matrix B and a skew-symmetric matrix C such that $A = B + C$.

(2) Prove that every skew-symmetric matrix of odd order is singular.

(3) Let

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Find A^{-1} using the adjoint method.

(4) Solve the following system by

- (a) The inverse method (i.e. by using the inverse of the coefficient matrix).
- (b) Gramer's rule.

$$2x_1 + 3x_2 + 2x_3 = 0.$$

$$2x_1 + 2x_2 + x_3 = 2.$$

$$3x_1 + x_2 + x_3 = 3.$$

(5) Find the determinant of the following matrix using cofactors:

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 9 & 2 & 3 & 2 \\ 8 & 2 & 2 & 1 \\ 7 & 3 & 1 & 1 \end{bmatrix}$$

- (6) Let x be an $n \times 1$ vector such that $x^T x = 2$. Find $(I_n - xx^T)^{-1}$.