

Assignment 2

Due: will be decided tomorrow)

Instructions: Show your work and explain every step. You may not be given credit at all for incomplete solutions. Answers with no explanations will not be accepted. Do not write the numbers in the decimal form (keep them as fractions). Note that I may grade only selected question(s) and only selected parts of the selected questions. Work on the assignment alone. Do not work in groups. The only person you can ask for help is me. You may get no credit at all if you cooperate on the assignment with someone else.

- (1) Determine if the each of the following sets of vectors form a basis for $\mathbb{R}^3$. Give full explanation. If the given set is a basis for $\mathbb{R}^3$, write the vector $(2, 3, 4)$ as a linear combination of the vectors in the basis:

(a) $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}.$

(b) $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}.$

(c) $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \right\}.$

(d) $\left\{ \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$

- (2) Determine if the following set of polynomials form a basis for P_2 . Give full explanation. If the given set is a basis for P_2 , write the polynomial $2+3t+4t^2$ as a linear combination of the polynomials in the basis:

$$\{3 + 2t + 2t^2, -1 + 2t + t^2, 3 + t^2\}$$

- (3) Decide if each of the following sets of vectors spans $\mathbb{R}^3$. Give full explanation. If the given set does not span $\mathbb{R}^3$, give an example of a vector in $\mathbb{R}^3$ that does not belong to the span of the given set:

$$(a) \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \right\}.$$

$$(b) \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \right\}.$$

- (4) Determine if the given set of polynomials is linearly independent in P_2 . Give full explanation. If the set is linearly dependent, write one of the polynomials in the set as a linear combination of the other polynomials in the set.

$$\{3 + 2t + 2t^2, -1 + 2t + t^2, 11 + 2t + 4t^2\}$$

- (5) Decide if the following set S is a subspace of the given vector space V :
- (a) $V = \mathbb{R}^4$. S is the set of all vectors in V of the form (a, b, c, d) , where $c = 3$ and $d = a - 3b$.
- (b) $V = P_2$. S is the set of all polynomials in V of the form $a + bt + ct^2$, where $a = b + c$.

- (6) Find a basis for $\mathbb{R}^3$ that includes the vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

- (7) Let $S = \{-1 + 2t + t^2, 2 + t - t^2, 5t - t^2, 5t + t^2\}$. Find a basis from S for $\text{span } S$.

- (8) Find a basis in M_{33} for all 3×3 skew-symmetric matrices.

- (9) Find a basis for the subspace of $\mathbb{R}^3$ consisting of all vectors of the form

$$\begin{bmatrix} 2b - a \\ 2a + b + 5c \\ a - b + c \end{bmatrix}.$$