

Assignment 4

Due: Friday, Dec 3, 04, at 2:00 pm in class

- (1) Determine whether the following are linear transformations or not:
 - (a) $L : P_2 \longrightarrow P_1, L(at^2 + bt + c) = (3 - b)t + (a - c)$.
 - (b) $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^3, L(x_1, x_2) = (x_1 - x_2 + 1, 0, -x_1)$.
 - (c) $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2, L(x_1, x_2, x_3) = (x_1 - x_3, 2x_2)$.
- (2) Let $L : P_3 \longrightarrow P_2, L(at^3 + bt^2 + ct + d) = (a - b)t^2 + ct$.
 - (a) Does $2t^3 + 2t^2 + 5 \in \ker L$?
 - (b) Does $2t^2 + 2t + 5 \in \ker L$?
 - (c) Does $5t^2 + 2t \in \text{range } L$?
 - (d) Does $5t^3 \in \text{range } L$?
 - (e) Find a basis for $\ker L$.
 - (f) Find a basis for $\text{range } L$.
 - (g) Find the matrix of L .
- (3) $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2, L(x_1, x_2, x_3) = (x_1 - x_3, 2x_2)$
 - (a) Find a basis for $\ker L$.
 - (b) Find a basis for $\text{range } L$.
 - (c) Find the matrix of L .
- (4) Find the coordinates of $(3, 4)$ with respect to the basis $[v_1, v_2]$, where $v_1 = (5, 4)$ and $v_2 = (4, 3)$. Also, find the transition matrix from $[e_1, e_2]$ to $[v_1, v_2]$ and the transition matrix from $[v_1, v_2]$ to $[e_1, e_2]$.
- (5) Let $v_1 = (5, 4), v_2 = (4, 3), w_1 = (3, 4), w_2 = (5, 7)$. Find the transition matrix corresponding to the change of basis from $[v_1, v_2]$ to $[w_1, w_2]$ and the transition matrix corresponding to the change of basis from $[w_1, w_2]$ to $[v_1, v_2]$.