

Homework 2

Remarks: Do not turn in this homework. These problems are for practice. Some of them may be assigned as an assignment and some will be done in class. More exercises may be added later. You have to show your work and explain every step. You may not be given credit at all for incomplete solutions.

Notation: If D is an $n \times n$ diagonal matrix whose diagonal elements are $d_1, d_2, \dots, d_n$, then D is sometimes written as $D = \text{diag}(d_1, d_2, \dots, d_n)$.

Definition: Let A be an $n \times n$ matrix. The *trace* of A , denoted $\text{tr}(A)$, is the sum of the elements of the main diagonal of A .

- (1) Let A be an $n \times n$ skew-symmetric matrix. Prove that if n is odd, then A is singular.
- (2) Let $D = \text{diag}(d_1, d_2, \dots, d_n)$. Give a necessary and sufficient condition for D to be singular.
- (3) Let $D = \text{diag}(d_1, d_2, \dots, d_n)$ and assume that $d_i \neq 0$, for $i = 1, \dots, n$. Find the inverse of D .
- (4) (True/False) If A is an $n \times n$ skew-symmetric matrix, then $\text{tr}(A) = 0$.
- (5) Let x be an $n \times 1$ vector such that $x^T x = 2$. Find $(I_n - xx^T)^{-1}$.
Hint: See P. 14 of Homework 1.
- (6) Let x be an $n \times 1$ vector. Is $(I_n - xx^T)$ invertible?
- (7) Find $\det(A)$, $\text{adj } A$, A^{-1} , the minor of a_{12} and A_{12} (the cofactor of a_{12}), where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ -2 & 3 & -1 \end{bmatrix}.$$

Do not write the numbers in the decimal form (keep them as fractions).

- (8) Solve the following linear system by using the inverse method (i.e. by using A^{-1} where A is the matrix of coefficients):

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 2. \\ 2x_2 + x_3 &= -3. \\ -2x_1 + 3x_2 - x_3 &= 4. \end{aligned}$$

- (9) Solve the following system by Cramer's Rule:

$$\begin{aligned} x_1 + 2x_2 &= 2. \\ -2x_1 + 3x_2 &= 4. \end{aligned}$$