

Homework 7

- (1) Find the eigenvalues of the following matrices. For each eigenvalue determine its geometric and its algebraic multiplicity. Also, find eigenvectors associated with the eigenvalues, and determine if the given matrix is defective. If the matrix is nondefective, find a matrix P that diagonalizes it.

(a)

$$\begin{bmatrix} 5 & -18 \\ 1 & -1 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}.$$

(c)

$$\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}.$$

(d)

$$\begin{bmatrix} 6 & -4 \\ 3 & -1 \end{bmatrix}.$$

(e)

$$\begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}.$$

(f)

$$\begin{bmatrix} 3 & -8 \\ 2 & 3 \end{bmatrix}.$$

(g)

$$\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}.$$

(h)

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}.$$

(i)

$$\begin{bmatrix} -8 & -9 & -12 \\ 2 & 1 & 4 \\ 2 & 3 & 2 \end{bmatrix}.$$

(j)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

(k)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(l)

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{bmatrix}.$$

(2) Find the eigenvalues of the following matrices. For each eigenvalue determine its algebraic multiplicity.

(a)

$$\begin{bmatrix} 8 & 5 & 6 & 0 \\ 0 & -2 & 0 & 0 \\ -10 & -5 & -8 & 0 \\ 2 & 1 & 1 & 2 \end{bmatrix}.$$

(b)

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 2 \\ -1 & -1 & 0 \end{bmatrix}.$$

(c)

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(d)

$$\begin{bmatrix} 4 & -5 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}.$$

(e)

$$\begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}.$$

(f)

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}.$$

(g)

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 6 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$