Assignment 3

(1) Let A be the matrix given below. Find a basis for the nullspace of A and the nullity of A (i.e. find a basis for the solution space of $Ax = 0$ and also find its dimension). Also, find two bases for the row space of A (one of them should be formed from rows from A and the other is not) and two bases for the column space of A (one of them should be formed from columns from A and the other is not). Find also the row rank of A, the column rank of A, and rank A.

$$\begin{bmatrix} 1 & 2 & 2 & -1 & 1 \\ 0 & 2 & 2 & -2 & -1 \\ 2 & 6 & 2 & -4 & 1 \\ 1 & 4 & 0 & -3 & 0 \end{bmatrix}.$$

(2) Let $v_1 = (2, -1, 1)$ and $v_2 = (-3, 1, 7)$.

(a) Find a vector in $\mathbb{R}^3$ that is orthogonal to v_1 and v_2. Call this vector v_3.

(b) Now let $S = \{v_1, v_2, v_3\}$. Then make S orthonormal and then write the vector $v = (-6, 7, 8)$ as a linear combination of the vectors in S.

(c) Is it possible to have an orthonormal set of 7 vectors in $\mathbb{R}^6$? Explain.

(3) Question 2 Section 6.8 (of the text).

(4) (Material of this question is not covered yet but will be covered Friday, so if you're planning on turning this assignment in Friday don't turn this question in.) Change the following basis S for $\mathbb{R}^3$ to an orthonormal basis using Gram-Schmidt process

$$S = \left\{ v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}.$$