

Homework 1

Do either:

- No. 5 and (one of No. 1 and No. 2) and (one of No. 3 and No. 4).

or

- No. 6.

or.

- No. 7 and No. 8.

1. Implement a one-string DTM, M , whose Σ is $\{\triangleright, \sqcup, a, b, c\}$ and which replaces a by b if a is preceded by c and a by c if a is preceded by b . In all other cases, M keeps the symbol as it is.
2. Implement a one-string DTM, M , whose Σ is $\{\triangleright, \sqcup, a, b, c\}$ and which replaces a by b if a is followed by c and a by c if a is followed by b . In all other cases, M keeps the symbol as it is.
3. Implement a one-string NDTM, N , whose Σ is $\{\triangleright, \sqcup, a, b, c\}$ and which accepts only the language $L = \{x \in (\Sigma - \{\sqcup\})^* \mid x \text{ does not contain at least one of } a, b, c\}$.
4. Implement a one-string NDTM, N , whose Σ is $\{\triangleright, \sqcup, a, b\}$ and which accepts only the languages

$$L_1 = \{(ab)^n(aba)^m \mid n, m \text{ are nonnegative integers}\}$$

and

$$L_2 = \{(aba)^i(ab)^j \mid i, j \text{ are nonnegative integers}\}.$$

5. Given the following one-string TM, M :

State k	input Symbol α	$\delta(k, \alpha)$
s	$\triangleright$	$(s, \triangleright, \rightarrow)$
s	0	$(q_0, \alpha, \rightarrow)$
s	1	$(\text{"no"}, 1, -)$
s	α	$(\text{"no"}, \alpha, -)$
s	$\sqcup$	$(\text{"no"}, \sqcup, -)$
s	β	$(q_2, \beta, \rightarrow)$
q_0	0	$(q_0, 0, \rightarrow)$
q_0	1	$(q_1, \beta, \leftarrow)$
q_0	β	$(q_0, \beta, \rightarrow)$
q_0	α	$(\text{"no"}, \alpha, -)$
q_0	$\sqcup$	$(\text{"no"}, \sqcup, -)$
q_1	0	$(q_1, 0, \leftarrow)$
q_1	α	$(s, \alpha, \rightarrow)$
q_1	β	$(q_1, \beta, \leftarrow)$
q_1	1	$(\text{"no"}, 1, -)$
q_1	$\sqcup$	$(\text{"no"}, \sqcup, -)$
q_2	β	$(q_2, \beta, \rightarrow)$
q_2	$\sqcup$	$(\text{"yes"}, \sqcup, -)$
q_2	α	$(\text{"no"}, \alpha, -)$
q_2	1	$(\text{"no"}, 1, -)$
q_2	0	$(\text{"no"}, 0, -)$

- Find the output corresponding to each one of the following input strings: 0011, 0110, 1101.
 - $(s, \triangleright, 000111) \xrightarrow{M^{15}} (q, w, u)$, find q , w , and u .
 - What is the language L that is accepted by M .
6. The following question has two parts:
- Implement a one-string DTM and a two-string DTM whose alphabet is $\Sigma = \{\triangleright, \sqcup, a, b, \alpha, \beta\}$ which accept only the language $L = \{a^n b^n \mid n \in \mathbb{N}\}$.

Notice that the input string x for the two-string DTM has to be on the first tape and the second tape has to be empty initially. You need to copy x on the second tape.

- Implement a one-string DTM, a one-string NDTM, and a two-string DTM whose alphabet is $\Sigma = \{\triangleright, \sqcup, a, b\}$ which accept only the language $L = \{(ab)^n \mid n \in \mathbb{N}\}$. Notice that the input string x for the two-string DTM has to be on the first tape and the second tape has to be empty initially. You need to copy x on the second tape.

7. Implement a one-string DTM and a two-string DTM whose alphabet is $\Sigma = \{\triangleright, \sqcup, a, b, \alpha\}$ which accept only the language $L = \{a^i b a^j \mid 0 \leq i \leq j\}$. Notice that the input string x for the two-string DTM has to be on the first tape and the second tape has to be empty initially. You need to copy x on the second tape.
8. Implement a one-string DTM and a two-string DTM whose alphabet is $\Sigma = \{\triangleright, \sqcup, 0, 1\}$ which accept only the language $L = \{x \mid x \text{ starts with } 10 \text{ and ends with } 10\}$. Notice that the input string x for the two-string DTM has to be on the first tape and the second tape has to be empty initially. You need to copy x on the second tape.