Definition 1. The time required by a Turing machine M on an input x is the number of steps to halting. If the machine does not halt; i.e. $M(x) = \nearrow$, then the time is ∞.

Definition 2. Let f be a function from the nonnegative integers to the nonnegative integers, then we say that M operates within time $f(n)$ if for any input x, the time required by M on x is less than or equal to $f(|x|)$.

Definition 3. **TIME($f(n)$)**: Complexity class of languages that are decided by multistring $(k-string)$ $(k-tape)$ Turing machines operating within time $f(n)$.

Definition 4. **NTIME($f(n)$)**: Complexity class of languages that are decided by $k-$string NDTM's operating within time $f(n)$.

Definition 5. Suppose that for a $k-$string Turing machine M_k and an input x,

$$(s, \triangleright, x, \triangleright, \epsilon, ..., \triangleright, \epsilon) \xrightarrow{M_k^*} (H, w_1, u_1, ..., w_k, u_k),$$

where H is one of the halting states. Then the space required by M_k on x is $\Sigma_{i=1}^{k}|w_i u_i|$. If M_k is with input and output, then the space required is: $\Sigma_{i=2}^{k-1}|w_i u_i|$.

Definition 6. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ and let M be a Turing machine; we say M operates within space bound $f(n)$ if the space required for any input x is less than or equal to $f(|x|)$.

Definition 7. SPACE($f(n)$): Languages decided by $k-$string deterministic Turing machines with input and output that operate within space bound $f(n)$.

Definition 8. NSPACE($f(n)$): Languages decided by $k-$string nondeterministic Turing machines with input and output that operate within space bound $f(n)$.

Definition 9. In the following, $k \in \mathbb{N}$.

1. P = TIME(n^k) = TIME($poly$) = $\cup_{j\in\mathbb{N}}$TIME(n^j).
2. NP = NTIME(n^k) = NTIME($poly$) = $\cup_{j\in\mathbb{N}}$NTIME(n^j).
3. EXP = TIME(2^{n^k}) = TIME(exp) = $\cup_{j\in\mathbb{N}}$TIME(2^{n^j}).
4. NEXP = NTIME(2^{n^k}) = NTIME(exp) = $\cup_{j\in\mathbb{N}}$NTIME(2^{n^j}).
5. PSPACE = SPACE(n^k) = SPACE($poly$) = $\cup_{j\in\mathbb{N}}$SPACE(n^j).
6. NPSPACE = NSPACE(n^k) = NSPACE($poly$) = $\cup_{j\in\mathbb{N}}$NSPACE(n^j).
7. L (or LOGSPACE) = SPACE($\log n$).
8. NL (or NLOGSPACE) = NSPACE($\log n$).

Definition 10. P**:** set of all languages decidable by Turing machines in polynomial times (i.e. TIME(n^k), $k \geq 1$).