

Questions about Recurrence Relations

For the following questions (*the questions are at the end*) , use only the following theorems/methods:

- Iteration.
- Let $r_1, r_2, \dots, r_k$ be distinct roots for the characteristic equation of a linear homogeneous recurrence relation with constant coefficients of order (*degree*) k . Then, the solution is given by (*assume the sequence is $\{a_n\}$*)

$$a_n = \beta_1 r_1^n + \beta_2 r_2^n + \dots + \beta_{k-1} r_{k-1}^n + \beta_k r_k^n.$$

- Let r be a root of multiplicity k for the characteristic equation of a linear homogeneous recurrence relation with constant coefficients of order (*degree*) k . Then, the solution is given by (*assume the sequence is $\{a_n\}$*)

$$a_n = \beta_1 r^n + \beta_2 n r^n + \dots + \beta_{k-1} n^{k-2} r^n + \beta_k n^{k-1} r^n.$$

- If we have distinct roots with different multiplicities, then we use a combination of the previous two theorems.
- To solve a linear nonhomogeneous recurrence relation with constant coefficients, first find a particular solution. Call this particular solution y_n . Then find the general solution of the corresponding homogeneous recurrence relation (i.e. do NOT determine the constants at this stage). Call this solution z_n . Now set $a_n = y_n + z_n$ and determine the constants (*of z_n*) by applying the initial conditions.

Remarks:

- By *solve the following recurrence relation*, we mean *find the explicit form*.
- If you are not given initial conditions, then what you'll get is the general solution.
- The initial conditions determine the constants that you end up with when you solve the corresponding homogeneous equation. Different initial conditions lead to different constants.

- I'll present in class tomorrow (Friday, Mar 23) another proof that shows that there are many problems with the proof technique some people were trying to use at the beginning of the semester. What I'll do will be directly related to recurrence relations.

Here are the questions.

Solve the following recurrence relations.

1. $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 6$, $a_1 = 8$, $n \geq 2$.
2. $a_n = 5a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
3. $2a_n + a_{n-1} = 6a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
4. $a_n - 5a_{n-2} = -6a_{n-4}$, $n \geq 4$. (*Here just find the general solution.*)
5. $a_n + 2a_{n-1} = -a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
6. $a_n - 2a_{n-1} = -a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
7. $a_n = 4a_{n-2} + 3n$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
8. $a_n = 5a_{n-1} - 6a_{n-2} + 2.5^n$, $a_0 = 6$, $a_1 = 8$, $n \geq 2$.
9. $a_n - 6a_{n-1} = 8a_{n-3} - 12a_{n-2}$, $n \geq 3$. (*Here just find the general solution.*)
10. $a_n - a_{n-1} = -12a_{n-3} + 8a_{n-2}$, $n \geq 3$. (*Here just find the general solution.*)
11. $a_{n-1}^2 = a_n a_{n-2}$, $a_0 = 1$, $a_1 = 2$, $n \geq 2$.
12. $a_n = \frac{a_{n+2} - 5a_{n+1}}{-6}$, $a_0 = 6$, $a_1 = 8$, $n \geq 0$.
13. Here find both a_n and b_n .

$$a_n = a_{n-1} + b_{n-1}, a_{n-1} = b_n + b_{n-1}, n \geq 1, a_0 = 1, b_0 = 2.$$
14. $a_n = 5na_{n-1}$, $a_0 = 1$, $n \geq 1$.
15. $a_n = 5^n a_{n-1}$, $a_0 = 1$, $n \geq 1$.
16. $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$, $a_0 = a_1 = 1$, $n \geq 2$.
17. $a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$, $a_0 = 8$, $a_1 = 2^{\frac{-3}{2}}$, $n \geq 2$.
18. $a_n^2 = 5a_n a_{n-1} - 6a_{n-1}^2$, $a_0 = 1$, $a_1 = 3$.
19. $a_n = 5\sqrt{a_n a_{n-1}} - 6a_{n-1}$, $a_0 = 1$, $a_1 = 9$.