

Name: SSN: Row:

Do **only** 4 of the following questions and show your work. Circle **clearly** the questions which you want to be graded. Circle **only** 4.

Question 1: (5 points) Let A be the interval $(-1, 2]$. Find $A \cap \mathbb{Z}$.

Solution: $A \cap \mathbb{Z}$ is the set of all integers included in the interval A . Notice that -1 is not one of them. Thus, $A \cap \mathbb{Z} = \{0, 1, 2\}$.

Question 2: (5 points) Let $\{a_n\}_{n=1}^{\infty}$ be defined by $a_n = 2^n$, $\forall n \in \mathbb{N}$. Let $S = \{1, 4\}$. Find $\sum_{i \in S} a_i$.

Solution: $\sum_{i \in S} a_i = a_1 + a_4 = 2^1 + 2^4 = 18$.

Question 3: (5 points) Find $\{a, b, \{a, b\}\} - \{a, b\}$ and $\mathcal{P}(\{a, b, \{a, b\}\} - \{a, b\})$.

Solution: $\{a, b, \{a, b\}\} - \{a, b\} = \{\{a, b\}\}$. Notice that $\{\{a, b\}\}$ is different than $\{a, b\}$.

$\mathcal{P}(\{a, b, \{a, b\}\} - \{a, b\}) = \mathcal{P}(\{\{a, b\}\}) = \{\emptyset, \{\{a, b\}\}\}$.

Question 4: (5 points) Let $X = \{1, 2, 3\}$. Give an example of a binary relation on X which is not symmetric and not antisymmetric at the same time.

Solution: Take $R = \{(1, 2), (2, 1), (3, 1)\}$. R is not symmetric because $(1, 3)$ is not in R and it's not antisymmetric because $(1, 2)$ and $(2, 1)$ are both in R and $1 \neq 2$.

Question 5: (5 points) Let R be the relation on $\mathbb{Z}$ defined by: $a R b$ iff $a \leq b + 5$. Prove by a counter example that R is not transitive.

Solution: Take $(10, 5)$ and $(5, 2)$. Both of them are in R , but $(10, 2)$ is not.