

Quiz #3

Name: SSN: Row:

Instructions: Do *ONLY* **four** of the following questions. **Circle** the questions which you want to be graded and **cross out** those which you do **not** want to be graded.

NOTE: *If you do not follow the instructions above, you may lose some points.*

Question 1: (5 points) How many six-digit decimal numbers with no repetitions start with 57 or 75?

Solution: $2 \cdot P(8, 4) = 2 \cdot 8 \cdot 7 \cdot 6 \cdot 5$.

Question 2: (5 points) If repetitions are not allowed, what is the number of seven-digit odd decimal numbers?

Solution: $8 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 5$.

Question 3: (5 points) How many eight-digit decimal strings read the same from either side?

Solution: 10^4 . Notice that if it is numbers instead of strings, then the answer would be $9 \cdot 10^3$.

Question 4: (5 points) Let $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z}$ be defined by

$$f(m, n) = m^2 - 2n + 5.$$

Prove by a counterexample that f is not one-to-one.

Solution: $f(4, 8) = f(2, 2)$.

Question 5: (5 points) In how many ways can 5 red (*identical*) balls and 7 blue (*identical*) balls be distributed into 20 distinct boxes with at most one ball to a box?

Solution: $C(20, 5) \cdot C(15, 7)$ or $C(20, 7) \cdot C(13, 5)$. Notice if there is no limit on the numbers of balls in a box, then the answer would be $C(24, 5) \cdot C(26, 7)$.

Question 6 (5 points) Find the coefficient of x^{13} in the binomial expansion of $(3 + 2x)^{20}$.

Solution: $(3 + 2x)^{20} = \sum_{k=0}^{20} C(20, k) 3^k (2x)^{20-k}$. We want $20 - k = 13$. This implies $k = 7$. Hence, the coefficient of x^{13} is $C(20, 7) \cdot 3^7 \cdot 2^{13}$.