# Quiz #2

(0 points) Name: ......................................        (0 points) SSN: ................

Do **All** of the following questions and show your work.

**Question 1:** (6 points) Let $A = \mathbb{Z} \times \mathbb{Z}$, and let $B = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$. Find $A \cap B$.

Solution: $\{(-1,0),(0,-1),(0,0),(1,0),(0,1)\}$.

**Question 2:** (7 points) Let $\{u_n\}_{n=1}^{\infty}$ be defined by

$$u_n = \tfrac{2}{3} - (\tfrac{1}{6})(\tfrac{1}{4^{n-2}}), \ \forall n \in \mathbb{N}.$$

Let $A = (-5, \tfrac{11}{3}] \cap [\tfrac{7}{4}, 5) \cap \mathbb{Z}^+$ and let $S = A - \{-4, 7\}$. Find

$$\Sigma_{i \in S} \ u_i$$

.

Solution: $A = \{2,3\}$. $S = \{2,3\}$. Therefore, $\sum_{i \in S} \ u_i = u_2 + u_3 = \tfrac{9}{8}$.

**Question 3:** (7 points) Let

$$A = \{(-1,2),(4,5),(0,0),(6,-5),(5,1),(4,3)\}$$

$$B = \{b \mid b = k^2 \text{ for some } k \in \mathbb{Z} \text{ and } (a,b) \in A \text{ for some } a\}.$$

$$C = \{x - 4 \mid x \in Z \text{ and } \tfrac{x^2 - 5x + 6}{-1576} \geq 0\}.$$

Find $(C \cup B) \cap \{\{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{0\}, -2, -1, \phi\}$.

Solution: $B = \{0, 1\}$. $C = \{-2, -1\}$. Therefore,

$$(C \cup B) \cap \{\{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{0\}, -2, -1, \phi\} = \{0, -1, -2\}.$$

**Question 4:** (6 points) Prove by mathematical induction:

$$(1 + 2 + 3 + ... + n)^2 = 1 + 2^3 + ... + n^3, \forall n \in \mathbb{N}.$$

Solution: First, prove that $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$, $\forall n \in \mathbb{N}$. We did that on Quiz 1. Or you can depend on the handout about arithmetic and geometric sequences. It is easy to see that the previously mentioned sequence is an arithmetic sequence with $a = d = 1$. Now depend on the formula for $S_n$ (the sum of the first $n$ terms). Then, replace $(1 + 2 + 3 + ... + n)^2$ by $\frac{n^2(n+1)^2}{4}$ and prove the new result by induction. (We did that as an example in class.)