Questions about the Pigeonhole Principle and Derangements

1. Prove that D_n is even iff n is odd.
2. Find a single value for the expression $\sum_{k=0}^{n} C(n,k)D_{n-k}$. Notice that $D_0 = 1$.
3. 300 people are to sit on 300 seats numbered from 1 to 300.
   - In how many ways, can those people sit.
   - Suppose now that all of those people decided to change their seats. In how many ways can they sit now?
4. Prove that if $n+1$ integers are chosen from the set $\{1, 2, ..., 2n\}$, then two of those chosen number differ by 1.
5. What is the minimum number of people in a city of 2 millions who share the same birth month and the same fisrt and second initials?
6. A college offers 521 classes. Classes can be held at 20 different times. What is the minimum number of classrooms needed?
7. A person has to choose 51 integers. The minimum has to be 1000 and the maximum 1099. Is it possible for that person to choose those 51 numbers so that no two of them are consecutive?
8. Show that if 5 integers are chosen from the set $\{1, 2, ..., 8\}$, then two of those 5 integers must add up to 9.
9. Suppose that $k_1 + k_2 + ... + k_n - n + 1$ objects are put into n boxes, where k_i, $i = 1, ..., n$ are positive integers and so is n. Prove that either the first box contains at least k_1 objects, or the second contains at least k_2 objects, ..., or the nth box contains at least k_n objects.
10. 17 persons have first names Chris, Eric, and Mike and last names Kincaid and Smith. Show that at least 3 persons mong those have the same first and last names and at least 6 of them have the same first name.
11. An inventory consists of a list of 115 items, each marked "available" or "unavailable". There are 60 available items. Show that there are at least 2 available items in the list exactly 4 items apart.
12. Don't forget the examples I gave in class today and the next group of questions. More questions will be addaed regularly.