CSCE 235 **Homework 4**

Due: Monday, April 16, 01

Show your answers and explain every step.

Question 1: Solve the following recurrence relation:

$$\frac{\sqrt{a_n}}{a_{n-1}} = a_{n-2}^4,\ n \geq 2,\ a_0 = 1,\ a_1 = 4.$$

Question 2:

Definition: Let $G = (V, E)$ be a simple undircted graph. Then, the complement of G, denoted $\overline{G}$, is the simple undirected graph whose vertex set is V and whose edge set is

$$\overline{E} = \{(u, v) \mid u \in V,\ v \in V,\ (u, v) \notin E\}.$$

In other words, $\overline{G}$ is a graph with the same vertex set as G and with an edge set of the missing edges (*only*) of G.

The following parts are unrelated. n and m are natural numbers.

1. What is the degree sequence of $K_{m,n}$?
2. How many edges does $\overline{K_{m,n}}$ have?
3. What is the degree sequence of $\overline{K_{m,n}}$?
4. How many edges does $\overline{C_n}$ have?
5. What is the degree sequence of $\overline{C_n}$?
6. Is there a simple undirected graph with a degree sequence 5, 2, 1, 1, 1?
7. Let G be a simple undirected graph. Is it possible for both G and $\overline{G}$ to have Euler cycles? If yes, write down an example.
8. Prove or disprove the following:

   If i and j are natural numbers, with $i < j$ and $i \geq 300$, then C_i is a subgraph of C_j.

Warning: *Do not consider particular values of n.*

Remark: *Denote the vertex set of C_i by $V = \{1, 2, ..., i\}$ and the vertex set of C_j by $V = \{1, 2, ..., j\}$. Notice that the numbering of the vertices for C_i and C_j must be consistent. In other words, the edge set of C_i is $\{(k, k+1) \mid k = 1, ..., i-1\} \cup \{(i, 1)\}$, and the edge set of C_j is $\{(k, k+1) \mid k = 1, ..., j-1\} \cup \{(j, 1)\}$.*

Question 3: All of the following parts are based on the following undirected simple graph $G = (V, E)$.

$V = \{1, 2, ..., 8\}$, $E = \{(6, 5), (5, 3), (5, 4), (4, 3), (3, 1), (3, 2), (1, 2), (1, 8), (2, 8), (8, 7), (7, 6)\}$.

1. Is G bipartite? Explain.
2. Does G have Hamiltonian cycles? If yes, mention one (*only one*).
3. Does G have Euler cycles? If yes, mention one. If not, explain why?
4. Is **(3, 4, 5, 3, 1, 2, 3)** a cycle in G? Is it a simple cycle?
5. Let H be the undirected simple graph with a vertex set $\{3, 4, 5, 2, 8\}$ and with an edge set $\{(3, 5), (3, 4), (2, 8)\}$.
   - Is H a subgraph of G? explain.
   - Is every simple path in $\overline{H}$ is also a simple path in $\overline{G}$? Explain.
   - Is every simple cycle in $\overline{H}$ is also a simple cycle in $\overline{G}$? Explain.
6. What is the degree sequence of G.
7. How many edges does $\overline{G}$ have? Explain.
8. Is there a path in G from vertex 1 to vertex 6 with no repeated edges and that includes all the edges of G and all the vertices of G? Explain.