Question 1: Prove the following by mathematical induction:

(a) $6.7^n - 2.3^n$ is divisible by 4, for every $n \in \mathbb{N}$.

(b) $(\frac{3}{2})^n \geq 1 + \frac{n}{2}$, for all $n \in \mathbb{N}$.

Question 2: Prove or disprove:

(a) $(p \longrightarrow q) \longrightarrow r \equiv p \longrightarrow (q \longrightarrow r)$.

(b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

Question 3: Prove or disprove:

(a) $x^4 - 4x^2 + 4$ is nonnegative for every real number x.

(b) $(n^2 - 5n + 6)^3 + (1 + (-1)^n)(2n + 1)$ is an even integer for every natural number n.

(c) The sum of every two different prime numbers is an even integer.

(d) If a and b are irrational numbers, then a(b+1)+b is an irrational number.

(e) The system

$$3x - y = 1$$
$$-6x + 2y = 7$$

has no real solution.

Question 4: Decide whether the following statements are true or false. Explain why.

(a) 8 is a prime number and 8 is an even integer $\longrightarrow$ either $\sqrt{5} = 1$ or $-2 > 0$.

(b) If the only y-intercept of $x^2 + 4y^2 - 1 = 0$ is 1, then the only x-intercept of $e^x - 4y - 3 = 0$ is $ln(3)$.