Graphs Practise Questions I

In the following questions, all graphs are undirected and simple.

Question 1:

Let $G = (V, E)$ be defined by $V = \{1, 2, ..., 200\}$, and

$$E = \{(i, j) \mid |j - i| \in (0, 3)\}.$$

Find $|E|$ and $\overline{G}$.

Solution:

Notice that for $3 \leq i \leq 198$, each vertex i is adjacent to verrices $i-2$, $i-1$, $i+1$, $i+2$. Vertex 1 is adjacent to vertices 2 and 3. Vertex 2 is adjacent to vertices 1, 3, and 4. Vertex 199 is adjacent to vertices 197, 198, and 200. Vertex 200 is adjacent to vertices 198 and 199. Thus, the number of edges of G is $\frac{10+196\cdot 4}{2} = 397$.

$\overline{G} = (V, \overline{E})$, where

$$\overline{E} = \{(i, j) \mid |i - j| \geq 3\}.$$

Notice that i and j have to be in V.

Question 2: Let $G = (V, E)$ be defined by $V = \{1, 2, ..., 200\}$ and

$$E = \{(i, j) \mid j > i\}.$$

Find the degree sequence of G and also find $\overline{G}$.

SOLUTION:

The degree sequence of G is: $\underbrace{199, 199, ..., 199}_{\textbf{200 times}}$**.**

$\overline{G} = (V, \overline{E})$**, where** $\overline{E} = \phi$**.**

Question 3: Let $G = (V, E)$ be defined by $V = \{0, 1, 2, ..., 41\}$, and

$$E = \{(i, j) \mid i \neq j \text{ and } i \text{ and } j \text{ are both even or both odd}\}.$$

Find $|E|$ and the degree sequence of G. Does G have an Euler cycle? What is $\overline{G}$?

Now let $H = (V', E')$ be defined by $V' = \{1, 2, ..., 20\}$, and

$$E' = \{(i, j) \mid i \neq j \text{ and } i - j \text{ is a multiple of } 4\}.$$

Is H a subgraph of G?

Now let $M = (V'', E'')$, where $V'' = V$ and $E'' = E \cup \{(1, 2)\}$. Does M have an Euler cycle? Is there a path from 1 to 2 with no repeated edges and that covers all edges and all vertices of M?

SOLUTION:

The degree sequence of G **is** $\underbrace{20, 20, ..., 20}_{\textbf{42 times}}$**.**

The number of edges of G **is 420.**

G **has no Euler cycles, because it is not connected.**

$\overline{G} = (V, \overline{E})$**, where**

$$\overline{E} = \{(i, j) \mid i + j \textbf{ is odd}\}$$

Yes, H **is a subgraph of** G**.**

M **has no Euler cycles, because vertex 1 and vertex 2 are of odd degree.**

Yes, there is a path from 1 to 2 with no repeated edges and that covers all edges and all vertices of M**, because** M **is connected and 1 and 2 are the only vertices of odd degree.**

Question 4: Let A be the adjacency matrix of a graph $G = (V, E)$, where $V = \{1, 2, ..., 5\}$. Column i of A corresponds to vertex i of G. A^2 is given by

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 2 \\ 2 & 4 & 2 & 1 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}$$

How may edges do G and $\overline{G}$ have? What is the degree of vertex 4? How many paths of length 2 are there from vertex 2 to vertex 3? What is the degree sequence of G and what is the degree sequence of $\overline{G}$?

Solution:

First notice that the ith element of the main diagonal of A^2 represents the degree of vertex i. Thus, $\delta(1) = 3$, $\delta(2) = 4$, $\delta(3) = 3$, $\delta(4) = 2$, $\delta(5) = 2$. Therefore, the degree sequence of G is 4, 3, 3, 2, 2 and the degree sequence of $\overline{G}$ is 2, 2, 1, 1, 0. Hence, G has $\frac{4+3+3+2+2}{2} = 7$ edges and $\overline{G}$ has 3. Notice that the number of edges of a graph is equal to half the sum of the degree sequence of that graph. Finally, the number of paths of length 2 from vertex i to vertex j is equal to the ijth entry of A^2. Thus, the number of paths of length 2 from vertex 2 to vertex 3 is equal to the 23th element of A^2, which is 2.

Question 5 Let $G = (V, E)$ be defined by $V = \{1, 2, .., 5\}$ and

$$E = \{(1, 2), (1, 4), (1, 3), (2, 4), (2, 3), (2, 5), (3, 5)\}.$$

Find the adjacency matrix A and the adjacency list representation of G. What does the row of A corresponding to vertex 3 represent? Now depend on A only to find the adjacency matrix of $\overline{G}$. Finally, graph G.

Solution: Will be done in class.