

Graphs Practise Questions I

In the following questions, all graphs are undirected and simple.

Question 1:

Let $G = (V, E)$ be defined by $V = \{1, 2, \dots, 200\}$, and

$$E = \{(i, j) \mid |j - i| \in (0, 3)\}.$$

Find $|E|$ and $\overline{G}$.

Question 2: Let $G = (V, E)$ be defined by $V = \{1, 2, \dots, 200\}$ and

$$E = \{(i, j) \mid j > i\}.$$

Find the degree sequence of G and also find $\overline{G}$.

Question 3: Let $G = (V, E)$ be defined by $V = \{0, 1, 2, \dots, 41\}$, and

$$E = \{(i, j) \mid i \neq j \text{ and } i \text{ and } j \text{ are both even or both odd}\}.$$

Find $|E|$ and the degree sequence of G . Does G have an Euler cycle? What is $\overline{G}$?

Now let $H = (V', E')$ be defined by $V' = \{1, 2, \dots, 20\}$, and

$$E' = \{(i, j) \mid i \neq j \text{ and } i - j \text{ is a multiple of } 4\}.$$

Is H a subgraph of G ?

Now let $M = (V'', E'')$, where $V'' = V$ and $E'' = E \cup \{(1, 2)\}$. Does M have an Euler cycle? Is there a path from 1 to 2 with no repeated edges and that covers all edges and all vertices of M ?

Question 4: Let A be the adjacency matrix of a graph $G = (V, E)$, where $V = \{1, 2, \dots, 5\}$. Column i of A corresponds to vertex i of G . A^2 is given by

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 2 \\ 2 & 4 & 2 & 1 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}$$

How many edges does G have? What is the degree of vertex 4? How many paths of length 2 are there from vertex 2 to vertex 3? What is the degree sequence of G ?

Question 5 Let $G = (V, E)$ be defined by $V = \{1, 2, \dots, 5\}$ and

$$E = \{(1, 2), (1, 4), (1, 3), (2, 4), (2, 3), (2, 5), (3, 5)\}.$$

Find the adjacency matrix A and the adjacency list representation of G . What does the row of A corresponding to vertex 3 represent? Now depend on A only to find the adjacency matrix of $\overline{G}$. Finally, graph G .