CSCE 235 **Exam III** **May 1, 01**

Name: ..... SSN: ..... Row: ....

Instructions:

1. Make sure that you have 6 different pages (with 4 questions).

2. If a question has choice, circle **clearly** the parts which you want to be graded. Make sure to cross out all parts which you don't want to be graded. If you don't finish a part and you don't want that part to be graded, then **cross it out**.

3. Show your work and explain your answers.

4. Do NOT use calculators or any electronic devices or anything related to the material.

5. Write down your name, SSN, row, and indicate left or right (*with respect to me*).

6. Cheating of any form will result in a grade of zero and in submitting your name to the Judicial Affairs.

7. Talking to any other student during the exam time is not allowed even if you finish early.

Question 1: Do only 3 of the following parts:

(a) (7 points) Solve $4x \equiv 4 \pmod{6}$.

(b) (7 points) Solve $4x \equiv 8 \pmod{80}$.

(c) (7 points) Solve $(4x + 4)(x + 3) \equiv 0 \pmod{6}$.

(d) (7 points) Solve the system:

$x + y \equiv 1 \pmod{4}$

$x + 3y \equiv 1 \pmod{4}$.

Question 2: Do only 3 of the following parts:

(a) (7 points) Let $a = -36$ and $b = 15$. Find the quotient and the remainder when a is divided by b.

(b) (7 points) Find gcd(36,25) and write it as a linear combination of 36 and 25.

(c) (7 points) Find the multiplicative inverse of 25 (mod 36).

(d) (7 points) Find 36 (mod -15).

Question 3: Do only 4 of the following parts. All graphs are simple and undirected.

(a) (8 points) Let G be a simple undirected graph. If the degree sequence of $\overline{G}$ is

4, 2, 2, 1, 1, 1, 1. What is the degree sequence of G?

(b) (8 points) When does the $n-cube$ have an Euler cucle? When does C_n have an Euler cycle if $n \geq 7$?

The $n-cube$ has an Euler cycle when .....

C_n, $n \geq 7$, has an Euler cycle when .....

(c) (8 points) Let $G = (V, E)$ be a simple undirected graph. Prove that if G is isomorphic to $\overline{G}$, then either $|V|$ or $|V| - 1$ is a multiple of 4.

(d) (8 points) How many edges does the $n-cube$ have? How many edges does $\overline{K_{19,35}}$ have?

Number of edges of the $n-cube$ is .....

Number of edges of $\overline{K_{19,35}}$ is .....

(e) (8 points) Let $G = (V, E)$ be the simple undirected graph defined by $V = \{v_1, v_2, v_3, ..., v_n\}$, $n \geq 100$, and

$$E = \{(v_i, v_j) \mid |i - j| = 1\}.$$

How many edges does G have? What is the degree sequence of G? How many edges does $\overline{G}$ have? Does G have a path from v_1 to v_n with no repeated edges and that includes all edges and all vertices of G? Explain.

(f) (8 points) Let G be a simple undirected graph with an adjacency matrix A. If the diagonal elements of A^2 are (not necessarily in order) 5, 1, 1, 2, 3, 4, 1, 1 , what are the diagonal elements of B^2, where B is the adjacency matrix of $\overline{G}$?

(g) (8 points) Let G be a simple undirected graph with an adjacency matrix A, where

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Without graphing, determine the degree sequence of G and the adjacency matrix of $\overline{G}$.

Question 4: Let $G = (V, E)$ be defined by $V_1 = \{1, 2, ..., 7\}$,

$$E_1 = \{(1,2), (1,7), (2,3), (2,7), (2,5), (3,4), (3,5), (4,5), (5,6), (6,7)\}.$$

Let $H = (V_2, E_2)$ be defined by $V_2 = \{2, 3, 5, 6\}$, $E_2 = \{(2,3), (2,5)\}$. Let $M = (V_3, E_3)$ be defined by $V_3 = \{1, 2, ..., 7\}$,

$$E_3 = \{(1,2), (1,7), (1,6), (2,7), (2,3), (2,5), (3,4), (4,6), (5,6), (6,7)\}.$$

(a) (5 points) Is H a subgraph of G?

(b) (5 points) What is the degree sequence of G?

(c) (6 points) Is G isomorphic to M? Explain.

(d) (5 points) Is G bipartite? Explain.

(e) (5 points) What is the adjacency matrix of G?