CSCE 235 **Exam II** **Mar 30, 01**

Name: .... SSN: .... Row: ....

Question 0: (2 points) Follow the instructions.

Question 1: Do only two of the following 4 unrelated parts.

(a) (7 points) Prove that $f: \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{Z}$, defined by

$$f(m,n) = m^2 - 2n + 5,$$

is not one-to-one.

(b) (7 points) Let $f: \mathbb{R} - \{2\} \longrightarrow \mathbb{R}$, be defined by

$$f(x) = \tfrac{3x-7}{x-2}.$$

Find f^{-1}.

(c) (7 points) Give an example of a bijective function from the interval $(3,5)$ onto the interval $(2,7)$.

(d) (7 points) Let $f: \mathbb{Z} \longrightarrow \mathbb{Z}$ be defined by

$$f(m) = m^2 - 5m + 6,$$

Prove that f is not onto.

Question 2: Do only two of the following 3 parts.

A group of 13 cs books and 7 math books are to be ordered on a (*regular*) shelf.

(a) (7 points) In how many ways can they be ordered if all cs books are distinct and all math books are distinct and if the math books are to be together?

(b) (7 points) In how many ways can they be ordered if all cs books are distinct and 3 of the math books are identical and the rest of the math books are distinct?

(c) (7 points) If all the books are dsitinct, in how many ways can they be ordered if no two math books are to be together?

Question 3: Do only two of the following 4 unrelated parts.

(a) (7 points) How many different 7-digit numbers can be formed out of the digits $\boxed{4,4,4,4,5,6,7}$?

(b) (7 points) How many different 6-digit octal numbers have distinct and nonzero digits?

(c) (7 points) How many 6-digit octal **numbers** read the same from either end. (*Warning: Notice that it is numbers not strings.*)

(d) (7 points) If repetition is not allowed, what is the number of 6-digit decimal numbers that contain a 2 and a 3?

Question 4: Do only two of the following 4 unrelated parts.

(a) (7 points) How many permutations of $\boxed{1234567}$ contain $\boxed{34}$ (*as a substring*)?

(b) (7 points) Suppose that someone wants to change the position of each one of the digits of the number $\boxed{1234567}$. In how many ways can that person do that?

(c) (7 points) In how many ways can 9 (*identical*) red balls and 13 (*identical*) blue balls be put into 22 distinct boxes at most one ball a box?

(d) (7 points) In how many ways can 9 (*identical*) red balls and 13 (*identical*) blue balls be put into 22 distinct boxes? (Note: here there is no limit on the number of balls in a box.)

Question 5: Do only two of the following 4 unrelated parts.

(a) (7 points) A 100-point exam consists of 7 questions. Each question has to be out of at least 5. In how many ways can the 100 points be distributed on the 7 questions?

(b) (7 points) Let $n \in \mathbb{N}$. Determine why the function $f : \{1, 2, 3, ..., n\} \longrightarrow \mathbb{N}$, defined by

$$f(k) = C(n, k)$$

is not one-to-one. *Warning:* It is wrong to consider particular values of n.

(c) (7 points) What is the minimum number of people in a city of 4 million who share the same birth month and the same birth day?

(d) (7 points) Prove that if $n+1$ distinct integers are chosen from the set $\{1, 2, ..., 2n\}$, then two of the chosen numbers differ by 1.

Question 6: Do only two of the following 4 unrelated parts.

(a) (7 points) Find the coefficient of $x^{23}y^{11}$ in the binomial expansion of $(2x + 3y)^{34}$.

(b) (7 points) Find the middle term in the binomial expansion of $(2 + 3y)^{34}$.

(c) (7 points) Prove that $\sum_{k=0}^{n} C(n, k)(-1)^k = 0$, $\forall n \in \mathbb{N}$.

(d) (7 points) Find the coefficient of $x^{k_1}y^{k_2}z^{k_3}$ in the binomial expansion of $(ax + by + cz)^n$, where k_1, k_2, k_3 are natural numbers whose sum is equal to n and a, b, c are nonzero real numbers. Use a generalization of the formula you'll get to derive a formula for the coefficient of ${x_1}^{k_1}{x_2}^{k_2}...{x_p}^{k_p}$ in the binomial expansion of $(a_1x_1 + a_2x_2 + ... + a_px_p)^n$, where a_i, $i = 1, ..., p$, are nonzero real numbers and k_i, $i = 1, ..., p$, are natural numbers whose sum is equal to n.

Question 7: Solve only two of the following recurrence relations:

(a) (7 points) $a_n = a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

(b) (7 points) $a_n = a_{n-1} + 6a_{n-2} + 2 * 5^n$, $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

(c) (7 points) $\sqrt{a_n} = \sqrt{a_{n-1}} + 6\sqrt{a_{n-2}}$, $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

(d) (7 points) $a_n = 7^n a_{n-1}$, $n \in \mathbb{N}$, $a_0 = 3$.