CSCE 235 **Exam #1** **Feb 16, 01**

Name: .... SSN: .... Row: ....

Question 1: The following parts are unrelated.

(a) (8 points) Prove by truth tables that $a \longrightarrow b \equiv \overline{a} \vee b$.

(b) Prove or disprove each of the following:

(i) (5 points) If n is a prime number, then n is odd.

(ii) (5 points) $3 + (-1)^n$ is an even integer for every natural number n.

Question 2: The following parts are unrelated.

(a) (6 points) Find $\{1, 2, \{1, 2\}\} - \{1, 2\}$.

(b) Decide whether the following are true or false:

(i) (4 points) $\{5\} \in \{2, 5\}$.

(ii) (4 points) $\phi \in \{2, 5\}$.

(iii) (4 points) If A and B are any sets, then $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

Question 3: The following parts are unrelated.

(a) (9 points) Let A be the interval $[-2, 2)$. Find $A \cap \mathbb{N}$.

(b) (9 points) Let A and B be nonempty subsets of a universal set U. Prove or disprove:

If $A \subseteq B$, then $\overline{A} \subseteq \overline{B}$.

Question 4: The following parts are unrelated.

(a) (9 points) Let a be the sequence defined by $a_n = (-1)^n$, $\forall n \in \mathbb{N}$. Is a increasing? Is it decreasing? Explain.

(b) (9 points) Let b be the sequence defined by $b_n = 2^{n-3}$, $\forall n \in \mathbb{N}$, and let $S = \{4, 7\}$. Find $\prod_{i \in S} b_i$.

Question 5: The following parts are unrelated.

(a) Let R be the relation on $\mathbb{Z}$ defined by:

$$aRb \text{ iff } a \leq b + 5.$$

(i) (5 points) Is R antisymmetric? If yes, prove it. If no, write down a counterexample.

(ii) (5 points) Is R symmetric? If yes, prove it. If no, write down a counterexample.

(b) (8 points) Let R be the relation on $\mathbb{Z} - \{0\}$ defined by:

$$aRb \text{ iff } ab > 0.$$

Find [3]. How many equivalence classes does R have? What are they?

Question 6: The following parts are unrelated.

(a) (8 points) Let R be the relation on $\mathbb{Z}$ defined by:

$$aRb \text{ iff } 3a + b \text{ is a multiple of } 4.$$

Is R transitive? If yes, prove it. If not, write down a counterexample.

(b) (10 points) Let R be the relation on $\mathbb{R}^2$ defined by:

$$(x, y)R(c, d) \text{ iff } x + 2y = c + 2d.$$

Find $[(1, 3)]$. What does $[(1, 2)]$ represent in the Cartesian plane?

Question 7: (18 points)

Prove the following formula by mathematical induction:

$$(2+4+6+8+...+2n)^3 = n^3(n+1)^3,\ \forall n \in \mathbb{N}.$$

Hint: It suffices to prove that

$$2+4+6+...+2n = n(n+1),\ \forall n \in \mathbb{N}.$$

WARNING: remember what I have told you in class. Use **ONL** the technique which I have used.

Question 8: (10 points) The following parts are unrelated.

(a) Let $X = \{e, f, g\}$. Give an example of a binary relation on X which is symmetric and antisymmetric at the same time.

(b) Let R be a relation on a set X. If $(a, b) \in RoR^{-1}$, prove that (a, a) and (b, b) are both in RoR^{-1}. Give full explanation and write down every step.