CSCE 235 | # Exam #I | **Feb 19, 01**

Name: .... SSN: .... Row: ....

Instructions:

1. Do the **question 8** and then do **ONLY** 5 of the first 7 questions.

2. Circle **clearly** questions which you want to be graded (rememeber everyone has to do question 8 and then 5 complete questions of the first 7 questions.) If you circle than the required number, then only the first 5 will be graded together with question 8.

3. Cross out any question which you don't want to be graded. If you don't finish a question and you don't want that question to be graded, then **cross it out**.

4. Show your work and explain your answers.

5. Do NOT use calculators or any electronic devices or anything which is related to the material.

6. Write down your name, SSN, and row.

7. Cheating of any form will result in a grade of zero and in submitting your name to the Judicial Affairs.

8. Talking to other students during the exam time is not allowed even if you finish early.

Question 1: The following parts are unrelated.

(a) (8 points) Prove: If a is a real number which is a multiple of 3, then $a^2 + 3$ is a multiple of 3.

(b) Prove or disprove each of the following:

(i) (5 points) If n is a natural number, then $(2n)^3 + (1 + (-1)^n)$is an even integer.

(ii) (5 points) If p is a prime number, then $p + 2$ is not a prime number.

Question 2: The following parts are unrelated.

(a) (6 points) Find $(\{1, \{2\}\} - \{1\}) \cap \{2\}$.

(b) Decide whether the following are true or false:

(i) (4 points) $\{5\} \in \{2, \{5\}\}$.

(ii) (4 points) $\phi \subseteq \{2, 5\}$.

(iii) (4 points) If A, B, and C are any sets, such that $A \subseteq B$, then $A \cap C \subseteq B \cap C$. and $B \subseteq A$.

Question 3: The following parts are unrelated.

(a) (9 points) Let A be the interval $(-2, 3] \cup [4, 6)$. Find $A \cap \mathbb{Z}$.

(b) (9 points) Let A, B, and C be nonempty subsets of a universal set U. Prove or disprove:

If $A \subseteq B$, then $B \cup C \subseteq A \cup C$.

Question 4: The following parts are unrelated.

(a) (9 points) Let a be the sequence defined by $a_n = n + (-1)^{n+1}$, $\forall n \in \mathbb{N}$. Is a increasing? Is it decreasing? Explain.

(b) (9 points) Let b be the sequence defined by $b_n = 1 + 2^{n-3}$, $\forall n \in \mathbb{N}$, and let $S = \{4, 7\}$. Find $\sum_{i \in S} b_{i-1}$.

Question 5: The following parts are unrelated.

(a) Let R be the relation on $\mathbb{Z}$ defined by:

$$aRb \text{ iff } a - b \leq 2.$$

(i) (5 points) Is R antisymmetric? If yes, prove it. If no, write down a counterexample.

(ii) (5 points) Is R symmetric? If yes, prove it. If no, write down a counterexample.

(b) (8 points) Let R be the relation on $\mathbb{Z} - \{0\}$ defined by:

$$aRb \text{ iff } 3ab < 0.$$

Find $[4]$. How many equivalence classes does R have? What are they?

Question 6: The following parts are unrelated.

(a) (8 points) Let R be the relation on $\mathbb{Z}$ defined by:

$$aRb \text{ iff } 3a + b \text{ is a multiple of } 4.$$

Is R symmetric? If yes, prove it. If not, write down a counterexample.

(b) (10 points) Let R be the relation on $\mathbb{R}^2$ defined by:

$$(x, y)R(c, d) \text{ iff } 2x + 3y = 2c + 3d.$$

Prove that R is transitive.

Question 7: (18 points)

Prove the following formula by mathematical induction:

$$5^n - 1, \text{ is a multiple of } 4, \forall n \in \mathbb{N}.$$

WARNING: remember what I have told you in class. Use **ONL** the technique which I have used.

Question 8: (10 points) The following parts are unrelated.

(a) Let $X = \{e, f, g\}$. Give an example of a binary relation on X which is an equivalence relation and a partial order at the same time.

(b) Let R be a relation on a set X. Prove that RoR^{-1} is symmetric. Give full explanation and write down every step.