

SS#

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Instructions: Do all of the following questions. Show your work and explain your answers.

Question 1: (20 points) Let $A = \{2\}$, $B = \{2, 3\}$, and $X = \{1, 2, 3, 4\}$. Find

(a) $(A \setminus B) \times B$.

(b) The power set of $A \cup B$.

(c) $(X \setminus A^c) \setminus A$.

(d) The power set of $A \times B$.

Question 2: (10 points) Use truth tables to prove that:

$(a \implies b)$ is equivalent to $(\sim b \implies \sim a)$. (i.e. $(a \implies b) \equiv (\sim b \implies \sim a)$.)

Question 3: (15 points) Let $A = \{1, 2, 3\}$ and define the following equivalence relation, R , on A :

$$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}.$$

(i) Is R antisymmetric? Explain.

(ii) Find the equivalence class of 3.

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Question 4: (15 points) Decide whether the following statements are true or false (i.e. prove or disprove). If they are false, then give a counter example.

(a) If a and b are irrational, then a^b is irrational.

(b) For any sets A , B , and C , if $A \subseteq B$, then $(A \cup C) \subseteq (B \cup C)$.

Question 5: (10 points) Use mathematical induction to prove that $10^{n+1} + 10^n + 1$ is divisible by 3 for all $n \geq 1$.

Question 6: (20 points)

(a) Give an example of a bijective function from the set $(\mathbb{N} \setminus \{1, 2, 3\})$ to the set $(\mathbb{N} \cup \{-1, 0\})$.

(b) Prove that the function $f : \mathbb{R} \longrightarrow (\sqrt{2}, \infty)$ defined by $f(x) = e^x + \sqrt{2}$ is bijective (i.e. one-to-one and onto.)

Question 7: (10 points) Draw the Hasse diagram of the poset $(A, \subseteq)$, where

$$A = \{\{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{1, 2, 4\}\}.$$

Extra Credit:

(1) (7 points) Let $A = \{1, 2, 3\}$. Give an example of a binary relation on A which is not symmetric and not antisymmetric.

(2) (7 points) Give an example of a bijective function from $\mathbb{R}$ onto the interval $(0,1)$.

(3) (7 points) Give an example of a poset which has no maximum element and no minimum element.