

Do **all** of the following questions and show your work. 1. Prove that the function $f : \mathbb{R} \longrightarrow \mathbb{R}^+$, defined by $f(x) = e^{2-x}$ is one-to-one.

2. Prove that the set $\mathbb{N} \setminus \{1, 2, 3, 4\}$ is countable by finding a bijective function from $\mathbb{N}$ onto this set.

3. Define the following relation R on $\mathbb{R}$.

$$(x, y) \in R \text{ if and only if } x + y = 77.$$

Is R reflexive? Symmetric? Antisymmetric? Transitive?

4. Let A , B , and C be nonempty subsets of a universal set U . Also, let $A \subseteq B$.

(a) Prove that $A \times C \subseteq B \times C$.

(b) Prove that $A \cup C \subseteq B \cup C$.