

Homework #1

Question 1: Let x be any integer. Determine whether the following statement is true or false. Give full explanation.

"If 5 is a solution of $e^{x-5} = 3$ or if $2x$ is an even integer, then $2x + 1$ is an odd integer and 4 is an even integer."

Question 2: Let $A = \{1\}$ and $B = \{1, 2\}$ be subsets of the universal set $X = \{1, 2, 3\}$. Find

(a) $A \setminus B$ (b) $A \cap B$ (c) $A \cup B$.

(d) $A \oplus B$ (e) $A \times B$ (f) A^c .

Question 3: Prove the following:

(a) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(b) $A \cup B = A \cup C$ and $A \cap B = A \cap C \implies B = C$.

Question 4: Prove or disprove

(a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

(b) $(A^c \cap (A \cup B)) \cup (A \cap B) = B$.

Question 5: Let $\mathbb{Z}$ be the set of integers and let q be a fixed positive integer. Define a relation R on $\mathbb{Z}$ such that $a \sim b$ if and only if $a - b$ is a multiple of q . Is R an equivalence relation on $\mathbb{Z}$? If yes, then what is $\bar{5}$?