Name: SS#

Instructions: Do all of the following questions. Show your work and explain your answers.

Question 1: (15 points) Solve the following conqruence system:

$$3x + 2y \equiv 2 \pmod{5}$$

$$x + 2y \equiv 3 \pmod{5}$$

Question 2: (15 points) Solve the following equation:

$$x \equiv 2x(3x+2) + 3 \pmod{7}$$

Question 3: (20 points) Let a and b be nonzero integers and let gcd(a,b) = g. Also, let m and n be integers satisfying:

$$g = ma + nb.$$

(a) Let M_1 and N_1 be integers satisfying:

$$gcd(a, -b) = M_1a + N_1(-b).$$

Express M_1 in terms of m and N_1 in terms of n.

(b) Express the least common multiple of -a and -b in terms of g, a, and b.

Question 4: (20 points) Solve the following recurrence relation by the method of $generating\ functions$:

$$a_n = 4a_{n-1}, n \ge 1, a_0 = 3.$$

Question 5: (20 points) Solve the following recurrence relation by the method of *charactaristic functions*:

$$a_n = 4a_{n-2} + 3^n, n \ge 2, a_0 = 1, a_1 = 2.$$

Question 6: (5 points)

Let N be an even natural number. Find an integer $r, 0 \le r < 4$, such that $1 + 2^{5N} \equiv r \pmod 4$

Question 7: (5 points) Find the coefficient of x^n in the expansion of $\frac{3}{1-4x^2}$.